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# Elastic deformation in thin film hydrodynamic lubrication

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ABSTRACT

An elastohydrodynamic numerical simulation is conducted for one-dimensional fixed slider plane bearings. The numerical model takes into account the piezoviscous effect of the lubricant and elastic deformation of the bounding surfaces to solve the one-dimensional Reynolds equation. It is found that a small elastic deformation of less than 100 nm plays an important role in load capacity in thin film hydrodynamic lubrication. As the film thickness decreases, a flat film shape appears from the leading side of the contact area. The expansion of the flat film thickness over the contact area leads to considerably lower load capacity.



## 1. Introduction

The pressure produced in a lubricant film is transmitted to the bounding surfaces, resulting in elastic deformation of the bounding surfaces. The elastic deformation can significantly change the geometrical shape between the bounding surfaces. It is apparent that in non-conformal contact where the applied load is supported the influence of the elastic deformation is significant because the fluid pressure increases up to several gigapascals. Conversely, it has been believed that in conformal contact, such as in journal bearings and thrust bearings, the elastic deformation can be ignored under low load because the fluid pressure is insufficient to cause large deformations of the surfaces. However, as the film becomes ever thinner it is possible that the elastic deformation becomes comparable with the film thickness to alter the film thickness distribution.

In the present study, an elastohydrodynamic (EHD) numerical simulation is conducted to investigate the influence of elastic deformation on conformal contacts. The model employed in the present study is a one-dimensional fixed slider plane bearing. The numerical analysis takes into account the piezoviscous effect of the lubricant and the elastic deformation of the bounding surfaces in solving the one-dimensional Reynolds equation. Numerical solutions for the elastohydrodynamic model are compared with those for an isoviscous rigid model.

## 2. Background

The importance of elastic deformation of the bounding surfaces was recognized during the development of the hydrodynamic lubrication theory for non-conformal contact such as that found in gears, rolling contact bearings, cams, and tappets. Martin [1] indicated that the film thickness predicted on the assumption of an isoviscous fluid and rigid surfaces for a line contact was too thin to explain the safe operation of gear teeth without any damage. Grubin [2] assumed that a flat film was formed in the contact area because the surfaces were elastically deformed by the fluid pressure of several gigapascals. He simply focused on the flow at the inlet zone, taking into consideration the piezoviscous effect of the lubricant. The film thickness equation suggested by Grubin [2] predicts the formation of fluid films of micrometer order, which is reasonable for gear contact conditions. Dowson and Higginson [3,4] numerically solved the Reynolds equation incorporating the elastic deformation and piezoviscous effect. The film thickness distribution obtained by Dowson and Higginson [3] had a flat shape in most of the contact area that was assumed by Grubin's model [2] and a constriction shape at the outlet.

As Grubin [2] and Dowson and Higginson [3] showed, the piezoviscous effect and elastic deformation are recognized as important factors in the EHD condition. Johnson [5] showed that in terms of both effects the hydrodynamic lubrication condition can be divided into four regimes as follows: isoviscous and rigid (IR), piezoviscous and rigid (PR), isoviscous and elastic (IE), and piezoviscous and elastic (PE).

For the PE regime, Grubin [2] and Dowson and Higginson [3,4] obtained numerical solutions. In the IE regime, the fluid pressure may be insufficient to change the viscosity of the fluid but can be

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**Nomenclature**

| | |
|---|---|
| $E_1$ | elastic modulus of moving surface (Pa) |
| $E_2$ | elastic modulus of stationary surface (Pa) |
| $E'$ | equivalent elastic modulus of moving surface (Pa) |
| $F$ | dimensionless frictional force $F=fh_0/(\eta_0 lu)$ |
| $G$ | bearing number $G=\eta u/(w/l)$ |
| $H$ | dimensionless film thickness $H=h/h_0$ |
| $K$ | convergence ratio $K=(h_1-h_0)/h_0$ |
| $P$ | dimensionless pressure $P=h_0^2 p/(6\eta_0 lu)$ |
| $Q$ | dimensionless mass flow rate $Q=q/(\rho_0 h_0 u)$ |
| $Q_c$ | dimensionless Couette mass flow rate $Q_c=q_c/(\rho_0 h_0 u)$ |
| $Q_q$ | dimensionless Poiseuille mass flow rate $Q_p=q_p/(\rho_0 h_0 u)$ |
| $S$ | dimensionless coordinate in direction of surface motion $S=s/l$ |
| $X$ | dimensionless coordinate in direction of surface motion $X=x/l$ |
| $W$ | dimensionless load $W=h_0^2 w/(6\eta_0 l^2 u)$ |
| $f$ | frictional force (N) |
| $h$ | film thickness (m) |
| $h_0$ | minimum film thickness (m) |
| $l$ | width of pad (m) |
| $p$ | pressure of fluid film (Pa) |
| $q$ | mass flow rate $q=q_c+q_q$ (kg/(ms)) |
| $q_c$ | Couette mass flow rate (kg/(ms)) |
| $q_q$ | Poiseuille mass flow rate (kg/(ms)) |
| $s$ | coordinate in direction of surface motion (m) |
| $u$ | sliding speed of moving surface (m/s) |
| $x$ | coordinate in direction of surface motion (m) |
| $w$ | load (N/m) |
| $\alpha$ | pressure–viscosity coefficient (Pa$^{-1}$) |
| $\delta$ | elastic deformation of moving surface $\delta=\delta_0-\delta_c$ (m) |
| $\delta_0$ | elastic deformation of moving surface (m) |
| $\delta_c$ | elastic deformation of moving surface at $x=\infty$ (m) |
| $\overline{\delta}$ | elastic deformation of moving surface (m) |
| $\rho_0$ | density at $p=0$ (kg/m$^3$) |
| $\rho$ | density (kg/m$^3$) |
| $\overline{\rho}$ | :dimensionless density |
| $\sigma$ | dimensionless parameter $\sigma=24\eta_0 l^2 u/(\pi E' h_0^3)$ |
| $\eta_0$ | viscosity at $p=0$ (Pa s) |
| $\eta$ | viscosity (Pa s) |
| $\overline{\eta}$ | :dimensionless viscosity |
| $\mu$ | friction coefficient |
| $\mu^*$ | dimensionless friction coefficient, $\mu^* = (l/h_0)\mu$ |
| $\nu_1$ | Poisson's ratio of moving surface |
| $\nu_2$ | Poisson's ratio of stationary surface |

enough to significantly deform the surfaces. The IE regime is found in contact between soft materials such as rubbers [6] and synovial joints [7]. In the IR regime, the pressure rise in the film is insufficient to cause the piezoviscous effect and elastic deformation as Martin [1] assumed for his solution. The hydrodynamic lubrication theory in the IR regime has been established in the literature [1,8–10].

In conformal contacts such as those operated in journal bearings and thrust bearing, the pressure generated in the film is lower than that in operation under the EHD condition but is enough to cause elastic deformation under high load. In the case of fixed slider bearings, the assumption of semi-infinite elastic bodies is reasonably applied in calculating the elastic deformation [11–16]. In the case of actual machine elements, a shell model [17–19] for a journal bearing and a beam model [20] for a pivoted bearing have also been used, because their surface deformations cannot be predicted on the assumption of semi-infinite bodies.

When the sliding speed is high, heat is generated in the film because of the viscous dissipation and is transferred to the surfaces. As a result, thermal expansion of the surfaces also occurs, which is sometimes of the same order as the elastic deformation [21–29]. The influence of thermal expansion becomes more pronounced than that of elastic deformation with increasing bearing size [21,22]. The thermal expansion and elastic deformation play important roles in pressure generation in tilting pad bearings under high load [23]. In particular, for parallel thrust bearings, Cameron [24] and Robinson and Cameron [25–27] suggested that a tapered surface because of heat expansion played a dominant role in pressure generation to support the applied load. Additionally, Baudry et al. [28] suggested that the supporting method of the pad influenced its distortion. Bennet and Ettles [29] suggested that a cantilever thrust bearing can form a wedge in the leading side of the pad by utilizing elastic deformation of the pad.

In the above studies, the focus was on the magnitude of the large elastic deformation and thermal distortion under high load and sliding. When the magnitude of the elastic deformation is small, the influence of the elastic deformation can be ignored. However, as the film thickness decreases, the magnitude of the elastic deformation becomes comparable with the film thickness even under low load, resulting in a change in film profile. To date, only a few researches [30–34] have dealt with small elastic deformations in thin film lubrication. Carl [30] described how elastic deformation cannot be ignored at a nominal pressure of about 7 MPa. Hemingway [31] stated that elastic deformation distribution depended on the location of the support ring for the pad and was responsible for producing pressure in a circular pad even under a nominal pressure of 2 MPa, as Baudry et al. [28] had shown in the high-load case. Nakamura et al. [32] investigated the influence of a small elastic deformation on hydrodynamic pressure generation in a parallel slide-way with a fluid film thickness of about 1.3 μm. They showed that the hydrodynamic pressure, which was built at the inlet zone, deformed the surfaces to significantly change the pressure profile. Yagi et al. [33] observed film thickness during the passage of micropits through a flat–flat contact in which a film thickness of about 200 nm was formed. They showed that even under a low nominal pressure of 0.2 MPa, lubricant was extracted from the leading edge of the micropits to deform the surfaces and cavitation occurred from the trailing edge of the micropits as the micropits entered the contact area. Better understanding of thin film lubrication has been anticipated.

## 3. Governing equations

Fig. 1 presents the schematic diagram of the fixed slider plane bearing in the present study. The contact area comprises a flat moving compliant surface with speed $u$ and a stationary inclined compliant pad of width $l$. The direction of motion is from left to right. Both surfaces are perfectly smooth. The coordinate $x$ is taken along the sliding direction with the origin located at the inlet. The initial maximum film thickness $h_1$ is located at the inlet ($x=0$) and initial minimum film thickness $h_0$ is located at the outlet ($x=l$). The lubricant flows only in the direction of the moving surface. The one-dimensional Reynolds equation is