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Analysis of the lubrication by a pseudoplastic fluid: Application to wire drawing

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ABSTRACT

The analysis of the plane isothermal flow of a thin pseudoplastic film with the viscosity index m is performed. The Reynolds equation related to the Newtonian case ($m=1$) is changed in a system of two equations, one giving the location of the zero shear stress, the other the pressure gradient. This system is applied to the analysis of the lubrication of the wire drawing with a cylindrical nozzle. We obtain so an analytical expression of the pressure Δp generated by the nozzle and the flowing thickness h_0. The approximate analysis based on the Reynolds equation overestimates Δp and h_0.

1. Introduction

For a pseudoplastic fluid, the shear stress τ is a power function of the shear rate $\dot{\gamma}$:

$$\tau = K|\dot{\gamma}|^{m-1}\dot{\gamma} \qquad (1)$$

The Newtonian fluid is the particular case where the exponent m is equal to 1, the strength K is in this case the fluid viscosity η; the limiting case $m=0$ corresponds to a rigid-plastic solid, with the maximum shear stress K. The Reynolds equation giving the pressure gradient in a thin Newtonian film allows the analysis of the hydrodynamic lubrication of pads, rollers… induced by the Newtonian fluids. The melted polymers [1], oils with polymeric additives, the metallic soaps such as sodium and calcium stearates [2] and the metallic materials in hot working [3] are pseudoplastic materials with $m<1$. Although some of these materials are used as lubricants especially the soaps in cold metal forming [4], the flow of a pseudoplastic thin film has been generally analysed only with approximations, with the exceptions of the Poiseuille flow and the pure rolling of two parts rolling at the same velocity [1].

In this article one establishes a set of equations describing the isothermal plane flow of a pseudoplastic thin film and applies them to the analysis of the lubrication of the wire drawing. Therefore before performing the analysis one briefly reviews this last subject.

2. The wire drawing lubrication

The wire drawing allows to reduce by cold plastic deformation the diameter $2R_i$ of a wire by drawing it through an axisymmetric die (Fig. 1). The wire/die friction must be minimised by the lubricant: drawing load increases with friction and too high friction can induce wire surface defects, the wire rupture and high die wear rate [4]. Therefore various experimental and theoretical works of wire drawing lubrication have been performed. Christopherson and Naylor [5] have demonstrated by experiments that a cylindrical nozzle, with a length L and leaving a radial gap h_p around the wire, placed before the die (Fig. 1) can increase greatly the pressure of an oil at the die entry and therefore promotes the hydrodynamic lubrication in wire drawing. With soap as lubricant Tattersall [6] observed that a nozzle increases the soap pressure at the die entry Δp and the flow rate Q lubricating the die. By assuming that the soap has a viscosity η_{ap} and by application of the Reynolds equation to the soap flow in the nozzle, he obtained the relations:

$$\Delta p = \frac{6\eta_{ap}u_0L}{h_p^2}\left(1-\frac{h_0}{h_p}\right) \quad \text{with} \quad h_0 = \frac{Q}{\pi R_i u_0} \quad \text{if} \quad h_p \ll R_i \qquad (2)$$

The viscosity η_{ap} so estimated starting from the experimental results decreases exponentially with the wire entry velocity u_0, a result due to the soap heating during its pressure increase according to Felder and Breinlinger [7]. In parallel, Walowit and Wilson [8] have performed the analysis of the lubrication by an iso-viscous Newtonian lubricant with the Reynolds equation. They have demonstrated that the flowing thickness h_0 related to the flow rate (2) is the lubricant film thickness at the point C of the entry of the work zone; at this point the pressure attains the

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Nomenclature

h	lubricant film thickness
h_p	radial gap between the wire and the nozzle
h_0	flowing thickness
h^*	coordinate where the shear stress vanishes
h_R	lubricant thickness where $h=h^*$
F,G	functions of H (Eqs. (9)–(10))
$H=h^*/h$	reduced coordinate where the shear stress vanishes
K	lubricant strength
L	length of the nozzle
m	strain rate exponent
p	pressure
R_i	initial radius of the wire
$R(m)$	numerical factor in the equation of Δp (Eq. (16))
$S(m,1)$	numerical factor in the equation of the flowing thickness (Eq. (20))
T	reduced lubricant film thickness (Eq. (14))
u	lubricant velocity
u_0	inlet wire velocity
x	coordinate along the film
y	coordinate across the lubricant film
Y	reduced lubricant film thickness (Eq. (11))
α	die semi-angle
Δp	pressure increase due to nozzle
$\dot{\gamma}$	shear rate
ε_y	sign of $(y–h^*)$
η	viscosity
τ	shear stress

Subscript

pn	pseudo-Newtonian

wire flow stress σ_0 and is maximum (Fig. 1). By taking into account the nozzle effect and for a conical die with semi-angle α (Fig. 1), they obtain

$$h_0 = \frac{3\eta u_0 \cot\alpha}{\sigma_0 - \Delta p} \quad \textit{at point C} \quad \textit{where} \quad \begin{cases} p=\sigma_0 \\ dp/dx=0 \end{cases} \tag{3}$$

The case of a piezo-viscous fluid can be easily deduced. Montmitonnet [2] and Montmitonnet et al. [9] have developed a more general but approximate approach that furnishes an estimation of the flowing thickness h_0 by taking into account the thermal effects and the pseudoplasticity of soaps. In this analysis called below “pseudo-Newtonian” [1,2] it is assumed that the fluid is Newtonian with a “mean” viscosity η_{pn}; this viscosity is constant in a slab of the film and is estimated starting from the mean value through the film thickness (along the coordinate y, Fig. 2) of the intensity of the shear rate of a Newtonian fluid:

$$\eta_{pn} = K\bar{\dot{\gamma}}^{m-1} \quad \text{with} \quad \bar{\dot{\gamma}} = \frac{1}{h}\int_0^h |\dot{\gamma}|\, dy \tag{4}$$

The strength K is estimated starting from the mean heating of the slab. The model is rather complex and only numerical results are given.

One does not know more recent theoretical work on the problem of the hydrodynamic lubrication in wire drawing. For the analysis of slider bearings lubricated with a power law fluid [10] and elastohydrodynamic lubrication with the same kind of fluid [11,12], a one dimensional modified Reynolds equation is derived for power law fluid, but it does not provide an explicit expression of the pressure gradient and to our knowledge it has not been applied to the problem of the lubrication in wire drawing.

Therefore our aim is to perform the exact analysis of the isothermal flow of a pseudoplastic fluid in wire drawing [13]. Plane flow is assumed because the film thickness is small with respect to the wire diameter. Indeed this analysis could be applied to other lubricated systems.

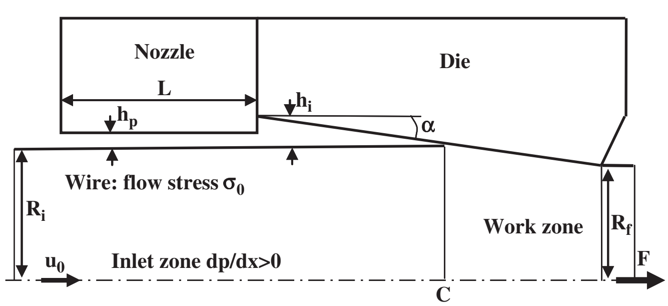

Fig. 1. The wire drawing process with a nozzle for promotion of the hydrodynamic lubrication and a conical die.

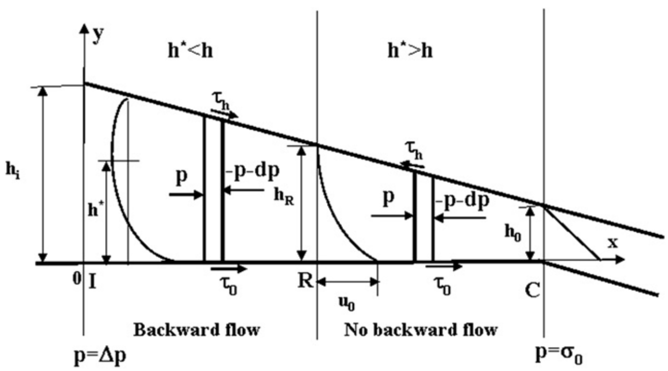

Fig. 2. Velocity field in the die and mechanical equilibrium of a lubricant slab.

3. Pressure gradient in the film

3.1. Local equilibrium equation and qualitative analysis of the velocity field

According to the usual assumptions of hydrodynamic lubrication, in the plane flow of a thin film, with velocity u along the axis x (Fig. 2), the shear rate and the equilibrium of a volume element read:

$$\dot{\gamma} = \frac{\partial u}{\partial y} \qquad \frac{\partial \tau}{\partial y} = \frac{dp}{dx} \tag{5}$$

One of the main problems for solving this equation set is due to the fact that in relation (1) there is the absolute value of the velocity gradient for $m \neq 1$. For the previously analysed cases, Poiseuille flow and pure rolling, this problem does not exist because the velocity gradient is always equal to zero at the film centre. By introduction of the position of the coordinate $y=h^*(x)$ where the velocity gradient is zero, integration of the equilibrium equation gives with (1):

$$K\left|\frac{\partial u}{\partial y}\right|^{m-1}\frac{\partial u}{\partial y} = \frac{dp}{dx}(y-h^*) \tag{6}$$