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Development of a texture averaged Reynolds equation

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ABSTRACT

The application of textured bearing surfaces results in a more complex lubricant flow pattern compared to smooth bearing surfaces. In order to capture the more complex flow pattern and possible inertia effects in the vicinity of the surface pockets, the Navier–Stokes equations should be used to model the flow between textured surfaces instead of the Reynolds equation. In this paper a multi-scale method is presented where the fluid flow in a single micro-scale texture unit cell is modelled using the Navier–Stokes equations, the results of which are then averaged to flow factors to be used in a novel texture averaged Reynolds equation on the macro-scale bearing level. Depending on the local flow conditions the non-linear inertia effects in the flow can either contribute or detract from the local load capacity of the lubricant film. Some results from the micro-scale calculations are presented, followed by the method developed to average these results to the macro-scale. The resulting flow-factors are presented and a load correction term is introduced. Although the method presented does not put restrictions to the texture dimensions, the texture unit cell dimensions are chosen equal to those in an experimental polymer water lubricated bearing. In a follow-up paper these results will be used to determine the efficiency of surface texturing in a lubricated journal bearing application.

1. Introduction

Hydrodynamic lubrication of textured bearing surfaces involves multiple scales. On the one hand the global or macro-scale, referring to the overall macroscopic geometry of the bearing system, and on the other hand the local or micro-scale, referring to the typical surface texture length scale. Such a multiscale approach was introduced into the field of tribology by Patir and Cheng [1,2] in which the surface roughness is considered at the local scale. Patir and Cheng's method is based on homogenization by averaging of the local scale flow. To describe the flow at the global scale, they introduced an average Reynolds equation with *flow factors*. Their method is widely known as the *flow factor method* and many slightly adapted versions have been published [3–5].

Experimental studies have shown that textured bearing surfaces exhibit a higher load carrying capacity compared to similar bearings with smooth surfaces (see for instance [6–9]). Two explanations for this additional load carrying capacity have been suggested in the literature: micro-cavitation phenomena and inertia effects. In near parallel sliding surfaces the formation of micro-cavitation at sub-ambient pressure in the dimples result in sucking in of additional fluid into the lubricating film which in turn results in an extra pressure build up [10–12]. However, the increase in load carrying capacity resulting from this mechanism is expected to be significant only in near parallel surface lubrication systems such as mechanical seals [8]. Furthermore, this mechanism is dependent on a sub-ambient cavitation pressure. In our study the focus is on the journal bearing application where the inlet suction mechanism is not significant. Therefore, micro-cavitation, that is cavitation at the texture level, is not included in the method outlined in this paper.

The validity of using the Reynolds equation in studying the effects of surface texture has recently been the subject of a number of studies, with as yet contradicting results [13]. Numerical analysis on single pockets, or dimples, has shown that convective inertia forces may play a role [14–17], such that the full Navier–Stokes equations apply to the local single dimple cell problem. For non-parallel bearings, the presence of dimples affects the pressure build up by the wedge effect as well, such that local microscopic analysis must be coupled to the macroscopic analysis in order to calculate the overall effect.

In our study the application of interest is the journal bearing, more specifically the water lubricated elastic surface journal bearing as it is used in marine propellor shaft support. Both textured journal bearings [9,18–20] and the use of elastic textured surfaces [21] have been the subject of recent publications, however, in our studies we combine these two. In this paper a novel *texture averaged Reynolds equation* is introduced with so called *texture flow factors*. In the classical flow factor method the

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Nomenclature

d_p	maximum pocket depth (m)
h	film height (m)
$\langle h \rangle$	average film height (m)
h_0	nominal film height (m)
$\overline{h}_t$	film height at global scale (m)
L	length of the unit cell (m)
p	pressure (N/m^2)
Q	flow (m^3/s)
R_p	pocket radius (m)
$\mathbf{u}$	velocity vector (m/s)
U_Δ	sliding velocity (m/s)
U	sum velocity (m/s)
U_1	lower surface velocity (m/s)
U_2	upper surface velocity (m/s)
(x,y,z)	Cartesian coordinate
z_s	pocket depth (m)
$\overline{\star}$	variable $\star$ referenced at the macro-scale
η	dynamic viscosity (Pa s)
ν	Poisson ratio (–)
ξ	pocket area ratio (–)
ρ	fluid density (kg/m^3)
$\chi_\mathbf{p}, \chi_\mathbf{s}$	flow tensor (–)
χ_{px}, χ_{py}	pressure flow factor (–)
χ_s	shear flow factor (–)
$\chi_{sp,x}, \chi_{sp,y}$	flow correction factor (–)

Reynolds equation is solved for the local unit cell problem. Since convective inertia forces may play a role [14,15,17,13] for a typical surface texture geometry (dimples), the full Navier–Stokes equations apply to the local cell problem. Therefore, in this paper the texture flow factors for a modified Reynolds equation as well as a pressure correction resulting from inertia forces are calculated by solving the Navier–Stokes equations on the local unit cell.

2. Mathematical model

The fluid pressure at a particular point in a lubricating film in a journal bearing depends on both the local and global film height variation. In particular at the bearing's thin film location, the geometry of the surface texture becomes important as local flow effects can generate non-symmetric pressure fluctuations due to convective inertia [14,15,17].

The bearing geometry, the surface texture and surface roughness each have their own characteristic lengths that differ by about two decades from one another. The large range of these characteristic lengths cannot be practically represented on a single level of detail. This means that one cannot represent details with a typical size of 0.1 mm at a bearing geometry with a typical size of 0.1 m without excessive computational costs. In this paper, we propose a texture averaged Reynolds equation that couples the surface texture scale in an averaged sense to the global scale that describes the bearing geometry. We assume that a clear distinction can be made between the characteristic length of these two scales. If this assumption holds, the length scales are *separable*, and a variable at the small scale can be represented on the larger scale in an averaged manner.

2.1. Texture geometry

In order to illustrate the averaging method for textured surfaces using the Navier–Stokes equations we limit ourselves to a two dimensional cell problem in this paper, although the method itself is valid for a three dimensional cell problem as well.

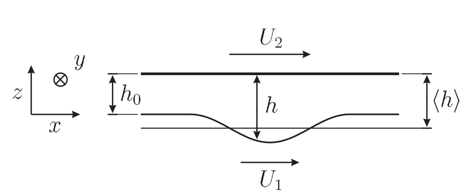

Fig. 1. 2D texture unit cell geometry. The nominal film height h_0 denotes the distance between the non-textured part and the counter surface.

The geometry of the two dimensional texture unit cell (TUC) is shown in Fig. 1. Analogous to Patir and Cheng's method, we define a local unit cell, capturing the texture geometry, and a corresponding smooth reference cell. The surface height of the textured surface is defined by

$$z_s = \begin{cases} -d_p \cos\left(\dfrac{\pi}{R_p}\dfrac{\sqrt{x^2}}{2}\right)^2 & \text{for } \sqrt{x^2} \le R_p \\ 0 & \text{for } \sqrt{x^2} > R_p \end{cases} \tag{1}$$

with d_p and R_p the maximum pocket depth and half the pocket width. The film height in the two dimensional texture unit cell is given by

$$h = h_0 + z_s \quad \text{with } h_0 > 0 \tag{2}$$

The nominal film height h_0 varies with the location in the global bearing system and its operating conditions.

2.2. Micro-scale: Navier–Stokes flow in the texture unit cell

Due to the large film height gradients that may occur for textured surfaces, convective inertia may possibly no longer be neglected [14,15]. The (linear) Reynolds equation therefore gives an incorrect description of the pressure distribution in the texture unit cell. The flow at the local scale is described by the (non-linear) Navier–Stokes equations. The instationary Navier–Stokes equations and continuity equation for an incompressible fluid with respect to a coordinate system fixed in space are

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \eta \nabla^2 \mathbf{u} \tag{3a}$$

$$\nabla \cdot \mathbf{u} = 0 \tag{3b}$$

with $D\mathbf{u}/Dt$ the material derivative, ρ the fluid density, $\mathbf{u}$ the velocity vector, p the pressure and η the viscosity. For the two dimensional case that is treated in this paper, the operator ∇^2 is defined as

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \tag{4}$$

For a single surface textured cell problem, it can be shown by a coordinate transformation that the cell problem for a moving texture in a moving reference frame reduces to the same steady state cell problem that holds for the stationary texture with respect to a stationary reference frame. For such a cell problem, the time-dependent part of the material derivative disappears from Eq. (3) and the steady state incompressible Navier–Stokes