Tribology International 43 (2010) 2139–2144

Contents lists available at ScienceDirect

Tribology International

journal homepage: www.elsevier.com/locate/triboint

# A comparison of multiaxial fatigue criteria as applied to rolling contact fatigue

M. Ciavarella [a,b,*], F. Monno [a]

[a] DIASS, Politecnico di BARI, V.le del Turismo, 8 (Q.re Paolo VI), 74100 Taranto, Italy
[b] CEMEC-PoliBA, Centre of Excellence in Computational Mechanics, V.le Japigia 182, Politecnico di Bari, 70125 Bari, Italy

ARTICLE INFO

*Article history:*
Received 23 October 2009
Received in revised form
26 May 2010
Accepted 8 June 2010
Available online 11 June 2010

*Keywords:*
Rolling contact fatigue
Plasticity
Shakedown
Hertzian contact

ABSTRACT

Under rolling contact fatigue (RCF) existing multiaxial fatigue criteria are not well validated and predict significantly different results. Results for simple typical Hertzian RCF pure rolling are shown as previously remarked by the authors, the Dang Van criterion applied to RCF gives over-optimistic fatigue limits, due to the large influence of the hydrostatic component of the stress, particularly under some conditions. It is here shown that the "simpler" Crossland criterion gives a more realistic fatigue limit of Hertzian peak pressure, and the more "elaborate" Papadopoulos criterion gives an even more conservative value, of about 3–3.5 times higher than the fatigue limit under pure shear. It is suggested that the multiaxial criteria per se do not give a reliable estimate of the fatigue limit, and perhaps an integration within Weibull-like theories should be attempted in the future, as well as a more "unified" approach and mix of criteria taken from gears design, rolling contact in railways, and in rolling bearings.


## 1. Introduction

A great number of mechanical applications suffer from rolling contact fatigue (RCF), including gears, bearings, and rail wheels. A lot of effort has been made to understand the forms of damage due to continuous rolling contact, and experimental evidence suggests that plastic flow (i.e. ratcheting) and high cycle fatigue are the most frequent forms of damage.

It is not entirely clear to date how far can the analogies between standard fatigue and rolling contact fatigue be extended, in particular concerning the interplay of plasticity damage and fatigue, or if there are too extensive differences to permit "technology transfer", and if the RCF is in turn very similar in railways, or gears or in rolling bearings. In an illuminating review of some of the recent work on the mechanism of rolling contact fatigue, Olver [1] tries to address some of these questions. Some analogies are clear, and basic mechanisms of RCF, as identified already by Littmann [2] from the perspective of the rolling bearings, and Way [3] with focus on rail–wheel contacts, are very close. Obvious differences are the lubrication conditions (generally better and more "controlled" in bearings), the presence of third bodies (including, for example, the role of water or leaves in railways), and the geometrical and loading conditions (again, more clearly defined and constants in bearings). From the material point of view, rolling bearings are manufactured in hard materials, with very high surface finish and of modest size, whereas rail steels are much softer and hence cause considerable plastic deformations before crack initiation. This has lead to significant development of a classical approach based on plasticity theory, shakedown and ratcheting, as described for example in Johnson's book [4, Chapter 9]. This approach is a little distant from corresponding approaches in fatigue (suggesting fatigue in RCF is quite different from our general understanding of standard fatigue), and in particular in the regime where ratcheting is activated (either significant friction, or sufficiently high normal loads) which has no equivalent for high number of cycles in standard fatigue. However, the original success of this approach seems today partly caused by coincidence and errors than by real features of the theory! In particular, very simple models of ratcheting by Merwin [5] and Merwin and Johnson [6] raised some hopes to model rolling contact fatigue by extremely simple ratcheting models, leading in other words to simple fatigue equations comparable to Basquin's law or Wohler curves. However, first problems emerged 20 years later when it was noticed that Merwin's calculation based on perfect plasticity were largely in error with refined FEM analysis [7], which found much higher ratchet rates than Merwin's, and prompted an improved elastic–perfectly plastic solution using distributed dislocations [8]. But perhaps more surprisingly, yet 20 years later, Ponter et al. [9] found that Merwin, while reporting his ratcheting results, had estimated the effect of cyclic hardening by an educated "a-posteriori" guess of the yield limit directly on the ratcheting results, rather than by other predictive methods or independent

* Corresponding author at: DIASS, Politecnico di BARI, V.le del Turismo, 8 (Q.re Paolo VI), 74100 Taranto, Italy. Tel.: +39 080 5962811; fax: +39 080 5962777.
*E-mail address:* mciava@poliba.it (M. Ciavarella).

0301-679X/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.triboint.2010.06.003

**Nomenclature**

| | |
|---|---|
| $a$ | half-width of the contact area |
| $p_{0-2D}$, $p_{0-3D}$ | line and point contact fatigue limit |
| $s(t)$ | deviatoric component of the stress tensor at the generic time step |
| $s_m$ | center of the smallest hypersphere circumscribing the load path in the deviatoric space |
| $\bar{s}(t)$ | deviatoric component of the mesoscopic stress tensor at the generic time step |
| $t$ | generic time step |
| $x$, $y$, $z$ | Cartesian coordinates |
| $\alpha$ | constant of the Papadopoulos criterion |
| $\alpha_{DV}$ | constant of the Dang Van criterion |
| $k$ | constant of the Crossland criterion |
| $\lambda$ | term of comparison of each criterion |
| $\sigma_e$ | fatigue limit under fully reversed bending |
| $\sigma_{eqv}$ | equivalent stress of the given criterion |
| $\sigma_{hyd}(t)$ | hydrostatic stress at the time step $t$ |
| $\sigma_{hyd,max}$ | maximum hydrostatic stress |
| $\tau_e$ | fatigue limit under fully reversed torsion |
| $\tau_{max}$ | maximum shear stress |
| $\sqrt{J_{2,a}}$ | amplitude of the square root of the second invariant of the stress deviator |
| $\sqrt{\langle T_a^2 \rangle}$ | the volumetric root mean square of the amplitude of the resolved shear stress |

measurement of hardening. Without this correction, the perfectly plastic prediction seems to largely over-predict the ratchet rate. Unfortunately, models of significant increased complexity like the Armstrong and Frederick non-linear kinematic hardening as that used by Bower and Johnson [10], despite leading to much better results, are still inadequate. The modeling of ratchet rate decay in rail steel is still very difficult, despite some progress with the very elaborate models [11,12]. Measurements on ratcheting anyway are only sparse, e.g. Clayton and Su [13] and Su and Clayton [14], and Tyfoor et al. [15], and considering this approach has been around for so long, it must be connected to some intrinsic difficulties in the measurements. Hence, the models based on failure as the ratcheting exhaustion of ductility [16], for both dry wear [17–22] and RCF [23] as plastic ratcheting processes, are still fascinating but questionable as quantitative methods. Indeed, it seem easier to interpret grossly RCF results with simpler hardness based empirical equations [24,25] such as those used in gear standards ISO [26] and AGMA [27] which are based on large number of experimental results.

If plasticity-based approaches have been attempted with partial success in rail–wheel contacts, where plastic deformations are certainly significant, most approaches in the field of rolling bearings, are based upon predictions of the elastic contact stress field [28–30]. However, even in the hardest steels some plastic deformation residual stresses and preferred orientations of grains develop even in bearing steels for high loads [31].

Olver also suggests that "running in" is a specific phase of RCF, with no equivalent in standard fatigue. It can be is beneficial, both in terms of wear because it mitigates fatigue by removing (at least, partially) cracks while they form, and because often residual stresses are compressive. Both these processes are however very difficult to quantify, and indeed Olver writes "*It would appear to be necessary to quantify these changes with some degree of confidence if reliable prediction of fatigue lives is to be achieved. This is not at present possible and represents a significant challenge to Tribology in the next* 35 *years*".

Various criteria exist for assessing the high cycle fatigue (HCF) of components under multiaxial conditions and the most used can be subdivided into three groups. In the first group, there are the *stress invariants* based criteria, which define an equivalent stress as an appropriate combination of the invariants of the stress tensor. The criteria belonging to the second group follow the *critical plane approach*. The fatigue crack is expected to nucleate on the plane where a given stress quantity reaches its maximum. Then, an equivalent stress is computed as the combination of stress related quantities evaluated on this plane. The third group consists in the *integral approach* criteria, which, unlike the previous ones, consider the damaging process as a combined effect of the state of stress on all planes. We shall use in this paper three criteria, namely those of Crossland, Dang Van and Papadopoulos. These criteria have been also compared with experimental findings, but mostly when the mean hydrostatic stress is positive (tensile) [32,33]. They differ more in some conditions than in others: for example, the Dang Van criterion suggests a very high phase difference effect in out-of-phase bending and torsion, whereas Papadopoulos' criterion suggests virtually no effect at all.

Recently, various authors [34–36], have proposed to use the Dang Van criterion [37] as a possible approach to one of the modes of failure for RCF problem, namely that corresponding to cracks initiated subsurface, but have found significant problems "*Results show that the usual technique for calibrating the constants of the Dang Van criterion does not agree with experimental evidence, especially for negative stress ratios*". Ref. [36] in particular suggested approximate equations, which however were found sometimes too crude approximation in Refs. [38,39], and more in details in Ref. [40]. It was found that the limit of the Dang Van criterion is significantly dependent on the Dang Van material constant, and for line contact the Dang Van limit becomes very high, falling in a region where ratcheting plastic deformations are expected. This also is in contradiction with classical experimental finding which suggests point contact to have about 75% higher RCF fatigue limit than line contact [41], perhaps because the regime above elastic shakedown in point contact is plastic shakedown rather than ratcheting. However, it is possible that very hard materials would be able to exploit the very high Dang Van limit "potential".

In the present note, we compare the Dang Van criterion for RCF with one integral approach criterion (Papadopoulos) and the more classical Crossland criterion. Some criteria are computationally expensive (Dang Van and Papadopoulos), but this computational costs seems not justified since the *results are surprisingly simple and a constant value can be extrapolated, at least in the absence of residual stresses*.

## 2. Rolling contact and fatigue criteria

First, the contact of a cylindrical roller on an elastic plane is considered. The hypothesis of plane strain state of stress is made, so that the problem can be analysed on a section plane perpendicular to the axis of rotation. Then, the contact of a rolling sphere is investigated. These are the simplest geometries in mechanical rolling components, and we can compare the theoretical results of our work with the experimental findings in [41].

As in Refs. [38,40], the origin of the Cartesian reference is taken on the point of contact, (Fig. 1) the $x$-axis lies along the direction