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Hydrodynamical modelling of lubricant friction between rough surfaces

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Abstract

The steady Couette flow of a Newtonian fluid between two plates, one of them a plane, the other one provided with riblets aligned perpendicular to the flow direction, is taken as a model for lubricant friction with wall roughness. In cases where the amplitude of the riblets is small compared to the riblet spacing, Reynolds lubrication approximation leads to an explicit solution. In contrast to this, a treatment of the full hydrodynamic equations is required for higher amplitudes. Under creeping flow conditions, an analytical treatment of the Stokes equations based on complex function theory allows for a reduction of the problem to the solving of ordinary differential and integral equations for functions of one variable. After this problem reduction, the resulting equations are solved by Fourier analysis and computer algebra.

The resulting streamline patterns of the flow reveal the formation of vortices under certain conditions. Since these vortices act like a kind of fluid roller bearings, their influence on the drag force and material transport of the lubricant is studied.

Keywords: Complex variable method; Lubrication approximation; Eddies; Drag reduction; Trapping of lubricant

1. Introduction

The laminar drag flow between two parallel plates, wellknown as plane Couette flow, is a standard example in hydrodynamics [1]. It is also the basis for the simplest model for lubricant friction [2,3], which predicts a linear dependence of the friction force F per plate area A on the velocity U of the moving plate according to

$$\frac{F}{A} = \mu_0 U, \qquad (1)$$

with the friction coefficient μ_0 given by the viscosity and the plate distance as $\mu_0 = \eta/H$. Modifications of plane parallel plates can also be found in literature e.g. flows with one plate being slightly inclined and of finite extension or Taylor–Couette flows with eccentric inner cylinder. The predominant part of these works is based on Reynolds' lubrication approximation [4]: for sufficiently thin films, i.e. if the aspect ratio L_z/L_x of the length scales is sufficiently small, the hydrodynamic equations are reduced to one single equation namely the Reynolds' equation [3]. Further requirements for the validity of the Reynolds' equation are, that the flow is steady, that the viscosity is constant, that inertia is negligible and that the film is yet thick enough to neglect the influence of Van der Waals forces on the flow.

In this paper we pay attention to the steady and incompressible Couette flow between two plates under creeping conditions with one plate being provided with a periodically varying profile. Note, that such plate corrugations—although unwanted—are often produced during manufacturing. Taking periodic plate corrugations also as a simple model for roughness, we especially investigate in cases where the amplitude of the profile is of the same order as its wavelength. Since the basic assumption $L_z/L_x \ll 1$ for Reynolds' lubrication approximation is violated in some cases, our analysis starts from the hydrodynamic equations, namely from the continuity equation, the Stokes' equations and the associated boundary conditions.

For an analytic description of the drag flow we make use of a methodical approach based on an exact solution of the biharmonic equation using complex function theory [5]. An important feature of this analytic approach is its exactness: in contrast to perturbation theory, it is not restricted to cases with small aspect ratio of bottom corrugations.

This paper is organized as follows: in Section 2 the system's geometry is defined. Field equations and associated

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boundary conditions are formulated. Using analytical methods, the mathematical problem is reduced to a set of integral identities for a single function of one variable in Section 3. Its solution is found by discretization in terms of Fourier series. Resulting streamline patterns are shown in Section 4 and compared with results from lubrication approximation. We analyze and discuss generation and evolution of vortices and their influence on drag and material transport.

2. General formulation

2.1. The hydrodynamical equations

The stationary flow of an incompressible Newtonian fluid between two plates, as shown in Fig. 1, is considered. The upper plate, moving with constant velocity U, is assumed to be plane, whereas the lower, fixed plate is provided with periodic corrugations of wavelength λ. The mean distance between the two plates is denoted by H. Gravity forces are not considered.

We use a Cartesian coordinate system with the x-axis placed at the mean level of the bottom contour and the z-axis oriented in vertical direction. The position $x=0$ is placed at a minimum of the undulation.

As characteristic length $\lambda/(2\pi)$ is used for length scaling in all directions. The velocity of the upper plate U is used for the scaling of velocities in all directions. For the pressure scaling we take the characteristic shear stress $2\pi\eta U/\lambda$, with η denoting the shear viscosity of the fluid. Using above scaling the upper and lower plate boundaries are given as

$$z = h, \tag{2}$$

$$z = b(x), \tag{3}$$

with the dimensionless mean plate distance $h{:=}2\pi H/\lambda$ and a 2π-periodic function $b(x)$ characterizing the shape of the corrugations at the lower plate. The mean value of b vanishes due to above definition of the coordinate system.

Neglecting inertia effects, the flow between the two plates is determined by the continuity equation and Stokes' equations

$$\nabla \cdot \vec{v} = 0, \tag{4}$$

$$\vec{0} = -\nabla p + \nabla^2 \vec{v} \tag{5}$$

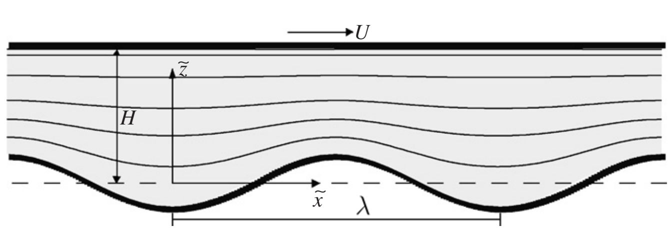

Fig. 1. Geometry and notations of the flow.

as basic field equations for the velocity field $\vec{v} = \vec{v}(x,z)$ and the pressure $p = p(x,y)$. Above equations are supplemented by the no-slip conditions at the lower plate $z = b(x)$ and at the upper plate $z = h$

$$\vec{v}(x, b(x)) = \vec{0}, \tag{6}$$

$$\vec{v}(x, h) = \vec{e}_x. \tag{7}$$

A two-dimensional flow geometry can be assumed, provided that the plates are of infinite length and depth. Then, the representation of the velocity field by a stream function $\psi = \psi(x,z)$ as

$$\vec{v} = \frac{\partial \psi}{\partial z}\vec{e}_x - \frac{\partial \psi}{\partial x}\vec{e}_z \tag{8}$$

is possible. Note, that by means of the ansatz (8) the continuity equation (4) is identically fulfilled. The Stokes equations (5) read then in terms of the stream function

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial z}\left\{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right\}\psi, \tag{9}$$

$$\frac{\partial p}{\partial z} = -\frac{\partial}{\partial x}\left\{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right\}\psi, \tag{10}$$

and the no-slip conditions (6), (7) take after decomposition in normal and tangential components the form

$$\left.\left\{\frac{\partial \psi}{\partial x} + b'(x)\frac{\partial \psi}{\partial z}\right\}\right|_{z=b(x)} = 0, \tag{11}$$

$$\left.\left\{\frac{\partial \psi}{\partial z} - b'(x)\frac{\partial \psi}{\partial x}\right\}\right|_{z=b(x)} = 0, \tag{12}$$

$$\left.\frac{\partial \psi}{\partial x}\right|_{z=h} = 0, \tag{13}$$

$$\left.\frac{\partial \psi}{\partial z}\right|_{z=h} = 1. \tag{14}$$

Eqs. (11), (13) are integrable and can be written alternatively as algebraic conditions

$$\psi(x, b(x)) = \psi_b = 0, \tag{15}$$

$$\psi(x, h) = \psi_s = \text{const}, \tag{16}$$

i.e. the shapes of the plates are streamlines.

2.2. Drag and material transport

We calculate the relevant integral quantities, namely the drag force and the flow rate. Subsequently by the brackets $\langle\cdot\rangle$ we denote the mean value over a period, i.e.

$$\langle f \rangle := \frac{1}{2\pi}\int_{-\pi}^{+\pi} f(x)\,\mathrm{d}x. \tag{17}$$