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Application of a simplified method to evaluate the inelastic state due to fretting fatigue

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Abstract

The simulation of fretting fatigue with the classical incremental method results in the lengthy and repeated calculations. This paper presents a simplified analysis method for the modeling of the mechanical behavior of inelastic state due to fretting fatigue. This approach has been proposed by Zarka et al. in order to predict the nature of the limit state of structures and the structural behavior under cyclic loading. It decreases significantly the computational complexity and duration of the calculations in comparison to classical incremental formulations.

This approach is applied to the problem of dry contact between cylider pads agians flat specimen. The calculations results are in good agreement with the experimental observations.

Keywords: Simplified analysis; Limit state; Non-proportional loading; Residual stresses; Fretting fatigue

1. Introduction

Fretting fatigue is a common and serious problem which occurs in a wide variety of engineering components such as in riveted joints or at contacting strands in wire ropes. It takes place when small oscillatory displacements between contacting materials cause surface damage. This eventually results in the development of a fatigue crack in components subjected to a superimposed alternating tensile stress. The prediction of damage and wear of materials under repeated moving contacts requires first the determination of the limit response in terms of stress and strain. The response of a structure subjected to a cyclic loading may be:

(a) *Elastic shakedown*: At any point of the structure, the plastic strain reaches a constant stabilized limit state (Fig. 1(a)).
(b) *Plastic shakedown*: For at least one point of the structure the plastic strain reaches a periodic stabilized limit state (Fig. 1(b)).
(c) *Ratchetting:* There is at least one point in the structure where the plastic strain rises incrementally until the collapse of the structure (Fig. 1(c)).

The incremental method based on a step-by-step integration could be used to calculate the possible stabilized state resulting in lengthy calculations. For a given load variation domain, safety factors against first yielding, inadaptation and limit state can be defined by the limit analysis and shakedown theories. Both these methods are limited. The former can take into account only the proportional loading states. The latter can be used with non-proportional stress state but it considers only elastic shakedown and as a result oversize the structure when plastic shakedown occurs. Other methods, called ''direct'' or ''simplified'' based on the shakedown theorems and their specialization to limit theorems are receiving increasing alternation for the prediction of structural failure in the inelastic range, although they are basically originated decades ago within classical plasticity. A survey of these methods is given by Weichert et al. [1]. In this paper, a simplified analysis method for the modeling of the mechanical behavior of inelastic structures is used to study the problem of fretting. This approach has been proposed by Zarka et al. [2] in order to predict the nature of the limit state of structures and the structural behavior under cyclic loading. It decreases significantly the computational complexity and duration of the calculations in comparison to classic incremental formulations. By means of a

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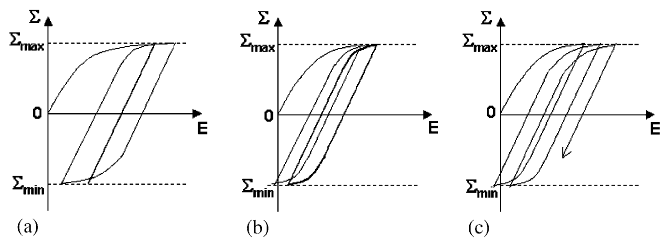

Fig. 1. Three limit states of structures.

decomposition of the real response (into an elastic and a plastic part) and of a change of variable, the plastic strains and the residual stresses can be determined in the whole structure, thanks to a purely elastic calculation. This method are used to study three-dimensional structures [3] and thin shell structures [4].

In a second part, application to fretting fatigue in relation with material fatigue properties are presented.

2. Principle of the method

The material considered is a standard material model [5] with the linear kinematic hardening assumption (Prager's model [6] with Von Mises yield criterion). These materials have an associate normal flow rule base on Drucker's postulate [7] and follow the maximal plastic work principle of Hill [8]. The assumptions of small deformations and quasi-static loading are made. Under these assumptions, the inelastic behavior of the materials is described by

- a yield function: $f(\sigma)\leqslant 0$,
- a normality rule: $\dot{\varepsilon}^{\rm p}=\lambda(\partial f/\partial\sigma)$,

where f represents one or several functions and defines the threshold associated with a unilateral constraint and with $\lambda\geqslant 0$ if $f(\sigma)=0$, and $\dot{\varepsilon}^{\rm p}=0$ if $f(\sigma)\leqslant 0$.

We denote $y(t)=CE^{\rm P}(t)$ the translation of the yield surface f. C stands for the hardening modulus and $E^{\rm P}(t)$ for the plastic strain. In the deviatoric stress space, f is a sphere. The center and the radius of this sphere are, respectively, $S(t)$ and σ_0. This sphere is defined by

$$f(S(t)-y(t))\leq\sigma_0^2, \tag{1}$$

where σ_0 stands for the yield stress, f for the Von Mises effective stress and $S(t)=\operatorname{dev}\Sigma(t)$ for the deviatoric part of $\Sigma(t)$.

The actual stress tensor $\Sigma(t)$ is decomposed as the sum of two terms

$$\sum(t)=\sum\nolimits^{\rm el}(t)+R(t), \tag{2}$$

where $R(t)$ is the residual stress field and $\Sigma^{\rm el}(t)$ is the elastic stress tensor of the structure assuming purely elastic.

The total strain is usually made of three components:

$$E(t)=E^{\rm e}(t)+E^{\rm P}(t)+E^{\rm I}(t), \tag{3}$$

where $E^{\rm e}(t)=M\Sigma(t)$ is the elastic part of the actual response and $E^{\rm I}(t)$ the initial strain of the structure. M is the elasticity matrix and it depends on E (Young's modulus) and υ (Poisson's ratio).

The inelastic response is represented by $R(t)$, $E^{\rm P}(t)$ and $y(t)$. In the deviatoric stress space, Eq. (2) is written as

$$S(t)=S^{\rm el}(t)+\operatorname{dev}R(t), \tag{4}$$

where $\operatorname{dev}R(t)$ denotes the deviatoric part of the residual stress tensor.

Eq. (4) is transformed to

$$S(t)-y(t)=S^{\rm el}(t)-(y(t)-\operatorname{dev}R(t)). \tag{5}$$

The simplified analysis introduces the modified structural hardening parameter $Y(t)$ which is defined as follows:

$$Y(t)=CE^{\rm P}(t)-\operatorname{dev}R(t). \tag{6}$$

This new tensor $Y(t)$ has no physical meaning, but it enables us to express the yield criterion in a new form which is based on the elastically computed deviatoric stresses

$$f(S^{\rm el}(t)-y(t))\leqslant\sigma_0^2. \tag{7}$$

We will now consider the Y space.

The fretting loading is non-proportional. In the Y space, the path of loading is represented by a series of spheres centered at $S^{\rm el}(t)$ with a radius of σ_0. If the intersection $C_{\rm L}$ of these spheres is non-void for all the points of the structure (Fig. 2(a)), the elastic shakedown occurs, if the intersection $C_{\rm L}$ is void for at least one point (Fig. 2(b)), then plastic shakedown occurs.

Moreover, we considered the structures submitted to fretting fatigue with high number of cycles. In other words, we suppose that the structure has reached a stable state of elastic shakedown (Fig. 1(a)).

Once the limit state is found, the plastic strains and residual stresses fields are determined by the following projection method. During one cycle, global elastic calculation is performed to determine $\Sigma^{\rm el}(t_i)$ for $t_i\in[0,T]$ and $i\in[0,N]$. In Y space, we obtain N spheres centered at $S^{\rm el}(t_i)$ with the radius of σ_0.

Since the limit state $Y_{\rm L}$ will be in $C_{\rm L}$ ($Y_{\rm L}$ must be plastically admissible at any time), the transformed parameter Y_1 after the end of the first cycle are locally projeted orthogonally on the local intersection $C_{\rm L}$ of the plastic yield surface, giving $Y_{\rm L}$.

To obtain better results, we defined an initial transformed parameter Y_0 obtained by a step-by-step resolution of the real elastoplastic problem. When Y_0 is known, we will obtain Y_1 by projecting Y_0 on the intersection of the first sphere $C(S^{\rm el}_{t_1})$ and the second sphere $C(S^{\rm el}_{t_2})$ (Fig. 3(a)). Then Y_1 is projected on the intersection of the second sphere $C(S^{\rm el}_{t_2})$ and the third sphere $C(S^{\rm el}_{t_3})$ to get Y_2