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A universal model for the load–displacement relation in an elastic coated spherical contact

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ABSTRACT

A finite element analysis was used in order to investigate the elastic contact of a coated sphere compressed by a rigid flat. Different coating and substrate geometrical and mechanical properties were analyzed to obtain a universal dimensionless model for the load–displacement relation. Both hard and soft coatings were considered. Dimensionless parameters, which control the behavior of the elastically loaded coated sphere, were identified. A proper normalization of the dimensional load and displacement was found resulting in a universal model. This model also provides a universal expression for the effective modulus of elasticity that is based only on mechanical properties of the coating and the substrate.

1. Introduction

Coatings are widely used in many applications to enhance tribological behavior of various contacting surfaces [1–5]. These include, for example, low wear and friction [6–8], better thermal and electrical conductivity [9,10] and increased resistance to plasticity [11], to name a few. In spite of their wide use the thicknesses of these coatings for best performance are still selected mainly by a trial and error approach. This is due to the lack of a universal model for predicting an optimal coating thickness that is based on a scientific theory.

Fig. 1 presents schematically the contact between a real rough surface and a rigid flat. As shown in the figure the rough surface is coated and the coating follows the roughness of the substrate. Contact occurs at the summits of the highest asperities, which according to the classical GW model [12] have spherical summits as can also be seen in Fig.1. Hence, the study of a single coated spherical asperity is needed for modeling the interaction between coated surfaces. A single asperity can either indent a mating elastic surface or be flattened by it [13]. Most of the theoretical studies of coatings found in the literature so far focus on the indentation of a coated flat substrate by an uncoated indenter, which is usually rigid and spherical [14–24]. Indentation is used mainly to characterize mechanical properties of coatings. In the present study we are interested in the behavior of an elastically loaded coated asperity where the mechanical properties of the coating are already known. Therefore, the flattening of the coated asperity by a rigid flat will be modeled in the present study (see also [11,25]). It should be noted here that, for good tribological design one should avoid asperities indentation and strive for asperities flattening that is associated with mild adhesive friction and wear.

Historically, the first model for the load–displacement relation in a homogeneous elastic spherical contact was provided by Hertz, see e.g. Ref. [26]. The Hertz solution for elastic contact, of a homogeneous sphere in contact with a rigid flat, provides the following relation between the contact load, P, and the corresponding displacement (interference) of the rigid flat, ω

$$P = \frac{4}{3}\frac{ER^{1/2}}{(1-\nu^2)}\omega^{3/2} \tag{1}$$

where E and ν are the Young's modulus and the Poisson's ratio of the sphere material, respectively.

Following the Hertz solution, several researchers attempted an extension of Eq. (1), where $P \propto \omega^{3/2}$, to include the effect of coating by introducing an effective modulus of elasticity E_{eff} that depends on the applied normal load (see Section 2 below). Doerner and Nix [14] used depth sensing indentation instruments to study the elastic modulus of coatings. They found that for small indentation depths relative to the coating thickness, E_{eff} approaches the elastic modulus of the coating material. For larger indentation depths E_{eff} approaches that of the substrate material. Hence, they suggested modeling E_{eff} as a summation of weighted contributions of the coating and the substrate. King [15] used the formulation for E_{eff} offered in Ref. [14] to numerically solve a punch indentation problem of coated substrates with flat ended cylindrical, quadrilateral and triangular punches. King [15] provided load–displacement relations for a number of punch shapes

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Nomenclature

E	Young's modulus
E_{eff}	effective modulus of elasticity
P	load
R	radius of the spherical substrate (inner radius of the coating)
R'	radius of the coated system, $R+t$
t	coating thickness
α	power in Eq. (6) (see Eq. (7))
λ	power in Eq. (2)
ν	Poisson's ratio
ω	interference
ω_{co}	interference contribution of the coating
ω_{su}	interference contribution of the substrate

subscripts

co	coating
su	substrate
t	transition

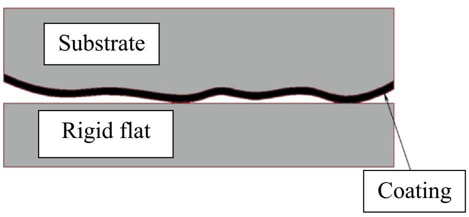

Fig. 1. A coated rough surface in contact with a rigid flat.

and sizes having various ratios of coating over substrate Young's moduli. In Ref. [16] a finite element method (FEM) was utilized to study elastic–plastic indentation of a coated substrate. The effective modulus of elasticity was calculated from the unloading curve and was found in good agreement with the empirical expression suggested in [14] and [15]. Bhushan and Venkatesan [17] used numerical approach to derive dimensionless empirical expression for the effective modulus of elasticity of a coated flat substrate indented by a spherical indenter. Their expression describes the variation of the effective modulus E_{eff} with respect to coating thickness, indentation depth and ratio of the elastic moduli of the coating over the substrate. A somewhat different approach was used in references [18,19] to extend the Hertz theory to the case of coated surfaces and to determine the mechanical properties of coatings. Richter et al. [20] used indentation tests to determine Young's modulus of cubic boron nitride coatings. Similar to the previous studies, the effective modulus of elasticity in [20] was also found to be load dependent. The authors suggest that this dependency is due to the absence of a proper correction to the influence of the substrate on the results. Zhao et al. [21] used FEM to study the indentation of a coated substrate by a conical indenter. The load–displacement relation as a function of the material properties of the substrate and coating, as well as the coating thickness, was established through extensive numerical simulations and curve fitting of the results. The load–displacement relation in [21] does not include an effective modulus of elasticity. In [22] nano indentation and pin on disk tests were used to characterize the elastic modulus and other parameters of TiN/Ti multilayer coating deposited on stainless steel substrate. Among other results it was shown that the modulus of elasticity decreases with the increase of indentation depth.

Garjonis et al. [27] used the classical Hertz solution, Eq. (1), along with FEM to investigate the elastic contact behavior of a sphere coated with a hard coating. The authors presented an effective modulus of elasticity of the coated sphere and showed that this effective modulus is mainly influenced by the interference. Yeo et al. [28] investigated a single spherical asperity of a coated rough surface. The study is focused on a hard coating and soft substrate and assumes that the entire asperity is made of the harder coating material. The individual stiffness values of the coating (asperity) and substrate were calculated separately and the concept of two springs in series was used to obtain an equivalent stiffness of the coated system. This stiffness, which is comparable to the effective elastic modulus in previous studies, also depends on geometrical and mechanical properties of the coating and substrate as well as on the interference.

As can be seen from the above literature review, the definition of the effective modulus of elasticity E_{eff} in a spherical coated contact is so far load dependent, which makes the load–displacement relation also load dependent, differently from the classical and elegant Hertz solution (see Eq. (1)). As will be shown in the following sections the current forms of E_{eff} are not only load dependent but also quite complex and involve different empirical constants that have to be obtained from experiments or numerical simulations. Such approach by itself prevents a universal model for the load–displacement relation. The goal of this study is to try a different approach that will provide a universal, load independent, model for the load–displacement relation similar to that by Hertz.

2. Theoretical background

2.1. Effective modulus of elasticity, E_{eff}

As stated in the introduction, when analyzing coated contact problems an effective modulus of elasticity E_{eff}, which accounts for the different elastic properties of the substrate and the coating can be very useful. It simplifies the analysis of such complex problems by using a single equivalent material property (e.g. [14,15,17]). It also helps to identify the modulus of elasticity of thin coatings from indentation tests (e.g. [19]). Another reason for using E_{eff} is to allow the extension of the classical Hertz solution for the relation between load and interference while maintaining the same format of Eq. (1) (e.g. [18,27]).

Two examples for the formulation of E_{eff}, are presented in Eqs. (2) and (3) that are taken from Refs. [14,17], respectively.

$$E_{eff} = \left\{ \frac{1-\nu_{ind}^2}{E_{ind}} + \frac{1-\nu_{co}^2}{E_{co}}\left(1-e^{\frac{-\lambda t}{\omega}}\right) + \frac{1-\nu_{su}^2}{E_{su}}\left(e^{\frac{-\lambda t}{\omega}}\right) \right\}^{-1} \tag{2}$$

$$\frac{E_{eff}}{E_{su}} = 1 + \left(\frac{E_{co}}{E_{su}} - 1\right)\exp\left[-\left(\frac{\omega}{t}\right)^m \left(\frac{E_{co}}{E_{su}}\right)^n\right] \tag{3}$$

In Eq. (2) the effective modulus, which is based on indentation tests, consists of weighted contributions of three components: the first term is the contribution of the indenter, the second is the contribution of the coating and the third is that of the substrate. The exponents in Eq. (2) depend on the coating thickness t, the interference (indentation depth) ω, and an empirical correction parameter λ, which accounts for different test conditions. More information regarding the range of the parameter λ depending on