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On the tangential contact behavior at elastic–plastic spherical contact problems

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ABSTRACT

The problem of tangential contact between an elastic–plastic sphere and a rigid plane is studied analytically and numerically with the specific aim to derive force–displacement relations to be used in numerical simulations of granular materials. The simulations are performed for both ideal-plastic and strain hardening materials with different yield stresses and including large deformation effects in order to draw general conclusions. The results are correlated using normalized quantities pertinent to the correlation of indentation testing experiments leading to a general description of the tangential contact problem. Explicit formulas for the normal and tangential forces are presented as a function of the tangential displacement using data that are easily available from axi-symmetric analyses of spherical contact. The proposed model shows very good agreement when compared with the FE-simulations.

1. Introduction

The mechanical problem of contact between two bodies is of substantial importance in many technical applications, for instance wear of machine components, indentation testing and compaction of powders, the latter feature being of most immediate interest presently. An accurate description of both the normal and tangential contact behavior is needed in order to derive reliable predictions in all of these subjects. However, the understanding of tangential contact, and especially tangential contact in combination with plasticity, is much less developed than for the corresponding case of normal contact.

The problem of normal contact of elastic bodies is well understood since the end of the nineteenth century from the Hertz contact theory [1]. The problem of contact between inelastic solids is much more involved and the development has mostly been driven by the interest of evaluating data from indentation testing. Two of the most important contributions regarding this feature were presented by Johnson [2,3] who found that the outcome of an indentation test can fall into three different regimes, level I, level II and level III, with different behavior of the normalized hardness $\overline{H}$ defined as

$$\overline{H} = \frac{F}{A\sigma_{rep}} \tag{1}$$

where F is the indentation force, A is the projected contact area and σ_{rep} is the yield stress at a representative value of the effective plastic strain, ε_{rep}. According to Tabor [4], $\varepsilon_{rep} = 0.2a/R$ where R is the radius of the indenter and a is the radius of the projected contact surface. Indentation tests of different materials can be correlated using a parameter Λ, at spherical indentation defined as

$$\Lambda = \frac{E}{(1-\nu^2)\sigma_{rep}}\frac{a}{R}. \tag{2}$$

A sketch of the behavior of the hardness as a function of Λ is shown in Fig. 1. In the level I regime, the elastic effects are dominant and contact can accurately be described by Hertz theory [1]. In the level III regime, elastic effects are negligible and the contact behaves in a rigid-plastic manner characterized by a constant normalized hardness. By utilizing self-similarity arguments, Biwa and Storåkers [5] and Storåkers et al. [6] were able to derive semi-analytical solutions for the contact force as a function of indentation depth. The derived relations have successfully been used in micromechanical analyses of powder compaction using both analytical [7–10] and numerical (discrete element) methods [11–15]. Finally in the intermediate regime, level II, neither elastic nor plastic effects can be neglected and self-similarity cannot be relied upon. However, a semi-analytical treatment for deriving force–displacement relations is still possible, Olsson and Larsson [16]. The understanding of the normal contact behavior in all three regimes has been further developed by extensive finite element calculations, pertinent to a wide range of materials, by Mesarovic and Fleck [17,18] and later Olsson and Larsson [19]. However, similar studies regarding tangential contact are rare in the literature. It should be noted that the

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Nomenclature

A	Contact area
a, a_0	contact radius, contact radius prior to shearing
E	Young's modulus of sphere
F	indentation force
F_N, F_N^0, F_N^{slip}	normal force on sphere, normal force prior to shearing, normal force at full slip
F_T, $\boldsymbol{F}_T$	tangential force, tangential force vector
f	correction function in calculation of δ_T^{slip}
f_N, f_T	normalized functions for describing the normal force and the tangential force
G	shear modulus of sphere
$\overline{H}$	normalized hardness
h, h_0	normal indentation, initial normal indentation
k_T, k_T^0	tangential stiffness, initial tangential stiffness
M, N	exponents in the description of $f_N(x)$
m	power-law hardening exponent
n	exponent describing $f_T(x)$
q, $\boldsymbol{q}$	contact shear stress, contact shear stress vector
R	radius of sphere
r	radial coordinate
x	normalized tangential displacement δ_T^{slip}
y	normalized parameter for calculating δ_T^{slip}
δ_T, $\boldsymbol{\delta}_T$	tangential displacement, tangential displacement vector
δ_T^{slip}	tangential displacement at full slip
ε	true uniaxial strain
ε_{rep}	representative strain
θ	circumferential coordinate
Λ	Johnson parameter
μ	Coulomb coefficient of friction
ν	Poisson's ratio of sphere
σ_Y, σ_0	initial yield stress, hardening parameter
σ	Cauchy uniaxial stress
σ_{rep}	representative yield stress

behavior in Fig. 1 is valid only for small deformations, at larger contact radii than $a/R > 0.1$, the normalized hardness drops due to large deformation effects.

When analyzing the tangential contact behavior, the problem becomes more complex due to the fact, first of all, that axi-symmetry does not apply and, secondly, the possible coupling between the contact pressure and the frictional shear stresses. However, for elastic contacts this coupling is generally small. Accordingly, by assuming that the normal pressure and the shear stresses are uncoupled, Cattaneo [20] and Mindlin [21] independently derived formulae for the tangential force as a function of the tangential displacement at constant and varying normal force. Further on, these authors found that the contact region is divided into two zones, in the center $0 \le r \le c$, the contact is in stick condition with zero relative displacement of the contact surfaces whereas in the outer area $c \le r \le a$ the contact is in slip condition where the shear stresses, $\boldsymbol{q}(r,\theta)$ is given by Amonton´s law as

$$\frac{||\boldsymbol{q}(r,\theta)||}{p(r,\theta)} = \mu \tag{3}$$

At increasing tangential load, the radius of the central stick zone decreases and when the norm of the tangential force $||\boldsymbol{F}_T||$ has reached the limiting Coulomb friction value μF_N, the whole contact area is in a sliding condition.

As mentioned earlier, tangential contact between two elastic–plastic bodies is far less studied (than the corresponding normal problem) in the literature and due to the complexity of the problem, any analytical treatment becomes problematic. Due to the size of the problem, it is only during the last few decades that it is has been possible to investigate tangential contact between spheres using the finite element method. The latter feature comes from the fact that axi-symmetry does not prevail and the problem

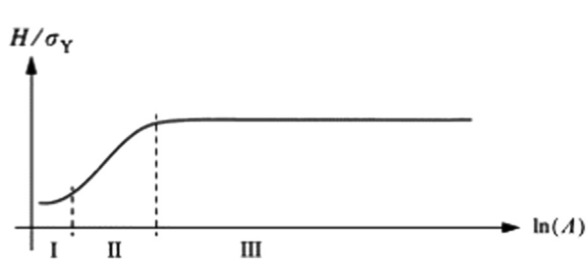

Fig. 1. Normalized hardness $\overline{H}$ as a function of ln Λ. The three levels of indentation responses I, II and III are also indicated.

of having a non-constant contact area necessarily leading to a dense mesh in a large region. However, as investigated by Larsson and Storåkers [22,23], when the material is described by power-law creep, the problem can, due to self-similarity, be reduced to the problem of a flat cylindrical punch indenting a half space. The benefit of using such an approach is that the contact region becomes stationary and thus the number of elements in the finite element mesh can be drastically reduced. However, the results derived by Larsson and Storåkers [22,23] are not directly applicable in the present case as elastic–plastic material behavior is at issue as well the influence from large deformations. Furthermore, it can be concluded that studies involving full 3D simulations of elastic–plastic tangential spherical contact [24–27] often focus on one specific type of material, for instance linear hardening or assuming that the whole contact area is in stick condition.

Accordingly, based on the above mentioned, the aim of the present work is to investigate how the tangential and normal forces varies with increasing tangential displacement at elastic–plastic contacts. In particular, the intention is to derive formulae that are relevant and useful for implementation of the elastic–plastic tangential contact behavior in a micromechanical analysis of powder compaction. The solution by Catteano [20] and Mindlin [21] forms the basis of the investigation but is presently extended to elastic–plastic contacts. Wear is an obvious area of application remembering that a large part of the results concerns the evolution of tangential and normal forces during sliding motion. In this study, the finite element method will be heavily relied upon due to the complexity of the problem.

2. Problem formulation

The problem studied in this work concerns a deformable elastic–plastic sphere with radius R, compressed against a rigid plane with a normal force F_N^0 resulting in an indentation depth of h_0 and a contact radius a_0. In a second step, a shear force F_T is applied while keeping the normal indentation h_0 constant, resulting in a tangential displacement δ_T. It is assumed that friction locally can be modeled with Coulomb friction, i.e. the maximum value of the shear stress, q, at the contact surface is given by $q = \mu p$ with p being the normal pressure. A sketch of the problem setup is shown in Fig. 2. Due to symmetry, this problem is also pertinent to contact between two equal spheres.