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A multi-Hertzian contact model considering plasticity

Michel Sebès [a,*], Hugues Chollet [a], Jean-Bernard Ayasse [a], Luc Chevalier [b]

[a] Université Paris-Est, IFSTTAR, GRETTIA, Champs-sur-Marne, France
[b] Université Paris-Est, MSME UMR 8208 CNRS, Champs-sur-Marne, France

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ABSTRACT

Multi-body-system (MBS) simulation is widely used in the railway industry. One of its major topics is the assessment of rolling contact fatigue (RCF). This damaging process is linked to plasticity. MBS wheel–rail contact models usually neglect plasticity as it does not change the vehicle behaviour. With the proposed method, contact stresses are consistent with a perfect plastic law. This new method has been recently detailed: it is an extension of the STRIPES semi-Hertzian (SH) model. A multi-Hertzian (MH) variant is here described, which is less exact but faster than the SH method. This new method has been implemented in a MBS package without resulting in a much longer execution time than elastic models.

1. Introduction

Multi-body-system (MBS) simulation has been used in the railway industry for decades. A key feature of railway MBS packages is the modelling of the wheel–rail contact [1]. In railway dynamics, one of the major subjects is the assessment of rolling contact fatigue (RCF). This damaging process is linked to plasticity.

In MBS packages, contact stress derivation is usually based on Hertz and Kalker's theories that both imply an elastic behaviour. Consequently, MBS calculations may provide unrealistically high stresses well above the yield limit.

The goal is here to derive contact stresses that are consistent with a perfect plastic law. A new model has been described in Sebès et al. [3]: it is an extension of the STRIPES semi-Hertzian model (SH) presented by Ayasse and Chollet [4], based on Kik and Piotrowski [10]. The scope of this paper is a multi-Hertzian (MH) variant, which is less exact but faster.

In Section 2, the MH method is described in its original form. The modifications in the MH method, in order to take plasticity into account are described in Section 3. The new model is benchmarked versus finite element method (FEM) in Section 4. Realistic application examples in wheel–rail contact are displayed in Section 5.

2. Multi-Hertzian (MH) method

2.1. Recall of Hertzian theory

The classical Hertzian contact is determined on the basis of the curvature ratio $\lambda = A/B$ with

$$A = \frac{1}{2}\frac{1}{R_{wyy}}, \quad B = \frac{1}{2}\left(\frac{1}{R_{wxx}} + \frac{1}{R_{rxx}}\right) \tag{1}$$

where R_{wyy}, R_{wxx} and R_{rxx} are the radii of wheel and rail. The curvature of rail around y is zero, x being the rolling direction. Radii are positive if the centre of curvature is inside the material. A *Hertzian geometry* meets following assumptions: constant curvatures and contact patch small compared to characteristic dimensions. The Hertzian ellipse has semi-axes a and b

$$a = m\left[\frac{3}{2}N\frac{1-\nu^2}{E}\frac{1}{A+B}\right]^{1/3} \tag{2}$$

$$b = n\left[\frac{3}{2}N\frac{1-\nu^2}{E}\frac{1}{A+B}\right]^{1/3} \tag{3}$$

where N is the force pressing the wheel against the rail in the z-direction; E is Young's modulus; ν is Poisson's ratio; n and m are Hertz functions of λ tabulated in [2]: their expression may be found in Ayasse and Chollet [4]. The aspect ratio of the Hertzian ellipse $\lambda_H = b/a$ is

$$\lambda_H = \frac{n}{m} \tag{4}$$

2.2. Interpenetration in a Hertzian geometry

Let h be the virtual interpenetration defined by

$$h = h_o - (z_w - z_r) \tag{5}$$

where z_w and z_r are respectively the wheel and rail profiles in the y–z plane. If profiles are in geometrical contact ($N=0$), z_w–z_r is zero at the contact point and positive elsewhere. The maximum interpenetration, h_o, is unknown in the problem. To find the proper value of semi-axis b in y-direction, h_o should be (Fig. 1)

$$h_o = b^2 B \tag{6}$$

* Corresponding author. Tel.: +33 1 81 66 87 07.
E-mail address: michel.sebes@ifsttar.fr (M. Sebès).

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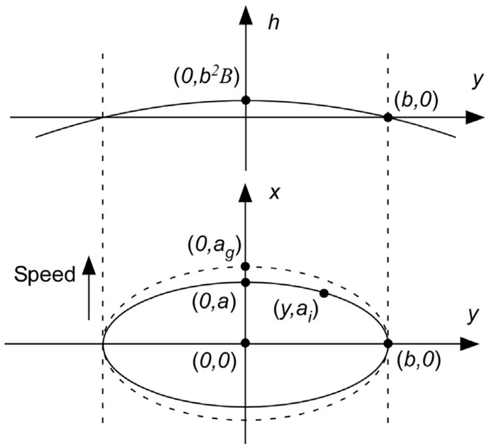

Fig. 1. Hertzian geometry – virtual interpenetration of profiles (top) and Hertzian and intersecting ellipses (bottom).

The intersection of the interpenetrated wheel in the rail is an ellipse. Its equation is given by setting $h=0$

$$Ax^2+By^2=h_o \tag{7}$$

The elastic Hertzian stiffness k_e, expressed in $N/m^{3/2}$, is deduced from expressions (3) and (6)

$$N=k_e h_o{}^{3/2} \quad k_e=\frac{2}{3}\frac{1+\lambda}{n^3}\frac{E}{1-\nu^2}\frac{1}{\sqrt{B}} \tag{8}$$

This expression enables to solve the normal problem with a multi-Hertzian method, abbreviated MH. Note that an important matter in a MH method is the selection of the number of potentially contacting ellipses. This selection is made a priori. If only one potential contact is chosen, this leads to wrong results in the case of multiple contacts. Conversely, if too many ellipses are selected, patches are likely to overlap, which is also unrealistic. This topic is developed in the following section.

2.3. Selection of potential ellipses

Rail profile (y_r, z_r) is discretised in strips: their location, angle and curvature $1/R_{rxx}$ are stored in tables. Let t_Y be the lateral position of the wheel relatively to the rail, t_Y being zero if the wheelset is centred in the track and flange contact occurring at a positive t_Y. For a given t_Y and a given strip, z_w-z_r, the relative vertical gap between the wheel and the rail, the wheel angle and its curvature $1/R_{wxx}$ are also tabulated. Every parameter used in Section 2.2 may be deduced from interpolation of tables over t_Y if the roll of the wheelset is neglected. It enables to solve the normal problem with the SH method as described by Ayasse and Chollet [4]. The same tables may be used to solve the normal problem with the MH method: the only change consists in considering interpenetration only at strips associated to ellipses and applying expression (8). The method of selection is based on the contact angle function: CAF [13,14]. Let γ be the contact angle in geometrical contact at a given t_Y: function $\gamma(t_Y)$ is called CAF. The following relationship between radii and γ is verified:

$$R_{wxx}+R_{rxx}=-\frac{1}{\cos\,\gamma(d\gamma/dt_Y)} \tag{9}$$

In a Hertzian geometry, radii are constant and so should be the term in the right side of previous expression. It follows that a discontinuity in CAF implies a change of ellipse: this criterion will be used. As an example, Fig. 2 shows CAF of the left wheel–rail profile of the Manchester benchmark [12]: 4 potential ellipses are

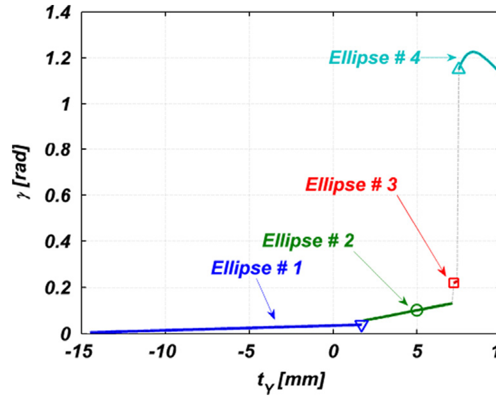

Fig. 2. CAF – left wheel–rail profile of Manchester benchmark.

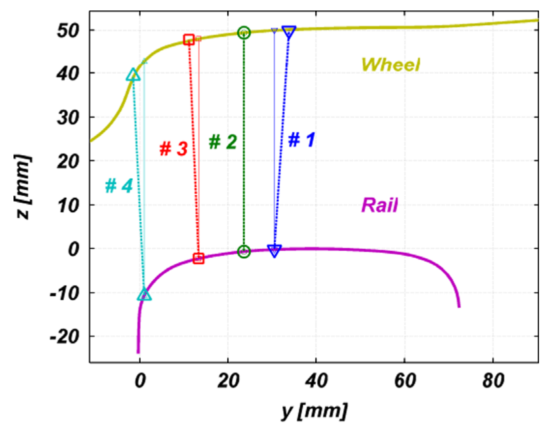

Fig. 3. Rail strips linked to ellipses if wheel is at t_Y=5 mm.

found with this criterion. Theoretical profiles exhibit locally constant radii: as a consequence, for low contact angles, expression (9) implies that the slope of CAF is nearly piecewise constant. The unique strip at which geometrical contact occurs at a given t_Y may be called the *main strip*. As an example, if the wheel is at t_Y=5 mm, the main strip will be associated to ellipse # 2 (circular marker in Fig. 2). Main strips linked to nearest t_Y of other ellipses (other markers of Fig. 2) will be considered for multi-contact. They are shown in Fig. 3.

2.4. Multi-Hertzian normal contact

In this section, subscript i is added to parameters in order to indicate they are linked to ellipse # i. Omitting tangent forces, the normal wheel–rail contact may be stated as follows: at a given wheel position t_Y, a vertical force Q being applied to the wheel, the vertical interpenetration δz between wheel and rail is searched in order to meet the following equilibrium:

$$\sum_i N_i\;\cos\;\gamma_i=Q \tag{10}$$

The vertical interpenetration δz between wheel and rail is linked to the normal one h_{oi} by a geometric relation

$$h_{oi}=(\delta z-(z_{wi}-z_{ri}))\cos\;\gamma_i \tag{11}$$

where $z_{wi}-z_{ri}$ are lengths indicated with thin vertical lines in Fig. 3, except an offset has been applied to ease its understanding: $z_{wi}-z_{ri}$ is zero at the main strip. If h_{oi} is negative, N_i is zero. Otherwise its value is given by expression (8), with subscript i added to every parameter. The difference in elapsed time between SH and MH methods lies only in the fact that summation (10) is