Wear 300 (2013) 96–104

Contents lists available at SciVerse ScienceDirect

Wear

journal homepage: www.elsevier.com/locate/wear

Stability criteria for a pyramidal shaped asperity ploughing through a plastically deforming substrate

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ARTICLE INFO

Article history:
Received 13 July 2012
Received in revised form
29 November 2012
Accepted 24 January 2013
Available online 8 February 2013

Keywords:
Abrasive wear
Adhesive wear
Galling
Contact models
Plasticity

ABSTRACT

In two body abrasion processes hard asperities plough through a soft surface. If the asperities can resist the forces that act on it, scratches will develop in the soft material. If the asperities cannot withstand these forces, they will break off and not cause direct abrasion damage. The same is the case for galling, where lumps develop on one of the surfaces because of material transfer. These lumps will abrade the counter surface, if the lumps are strong enough to withstand the forces that act on it. In order to describe these phenomena, simple criteria are desired to describe the mechanical stability of asperities and lumps.

In this work, an analytical model is presented for the mechanical stability of asperities. In the analysis, a pyramidal asperity shape will be assumed. Given the pyramidal asperity shape, several cases will be studied: the load is carried by a pyramid with a triangular base, a pyramid with a triangular base and an extended backside and the case where a crack has developed. Based on these models stability criteria of ploughing pyramidal asperities will be developed. Important results of the model will be discussed in the context of abrasion and adhesive wear processes.

1. Introduction

Two body abrasion is a very common wear process in which harder asperities plough through a softer surface. Also in some adhesive wear processes harder asperities plough through a softer counter surface. Here, in time, ploughing asperities can grow and develop into larger lumps due to adhesion or mechanical locking of soft material into the surface roughness of the harder surface. The transferred material work hardens and can cause scratches in the countersurface. In particular when high lumps can develop, scratches will be formed on the product due to abrasion. An example of such an adhesive wear process followed by subsequent abrasion, is galling in a deep drawing process. In the case of galling, the geometry of the developing lumps will determine the depth and width of the scratches which develop due to ploughing. Because galling can be detrimental for the surface quality of the products being made, it is important to control it in industrial practice. The main difference with two body abrasion is that in this case the shape of the ploughing asperity is not fixed beforehand, but dependent on the growth behaviour of the transferred material on the asperity.

Modelling of abrasive wear has often started with analysing a single asperity ploughing through a soft and flat substrate. Analysis of single ploughing asperities is then extended to rough surfaces by summing up these unit events, see e.g. [1,2]. In such models, single asperity behaviour as discussed above is typically used and applied to multi asperity contacts. Further, it is typically assumed that the asperities are rigid, so strong enough to withstand the forces that act on them during ploughing. Several reasons exist which limit the validity of the assumption of a rigid asperity and therefore neglect failure of the asperity. Single asperity ploughing has been extensively studied in experiments as well as in models. An overview of many studies is given in [3]. Much of the work on ploughing asperities has been restricted to 2D situations. Important is the work described in [4] where ploughing of 2D wedges is modelled by means of slipline models. Using these models, three slipline fields have been defined, identified with the names: wave formation, wave removal and cutting. Depending on the attack angle of the wedge and the strength of the interface between the wedge and the deforming material, the wedge will operate in a certain mode. The transitions between these regimes have been related to wear modes of spherical asperities by Hokkirigawa and Kato [5]. There, on the basis of experiments and a comparison with the slipline models of Challen and Oxley [4] a wear mode diagram is constructed in which three wear modes are distinguished: ploughing, wedging and cutting. This diagram is schematically depicted in Fig. 1 and

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http://dx.doi.org/10.1016/j.wear.2013.01.119

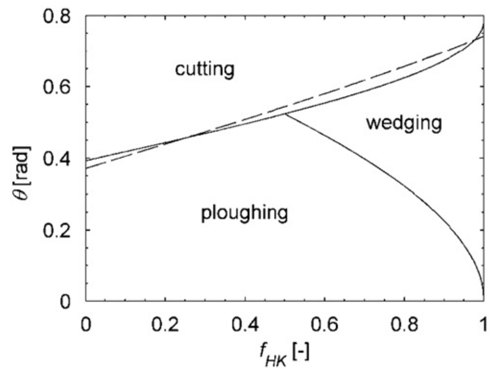

Fig. 1. Wear mode diagram according to Hokkirigawa and Kato [5].

will be discussed further. The transitions between the cutting regime and the other regimes are approximately given by the following equations, see also [6]:

$$\theta = 0.25(\pi - \arccos f_{HK}) \tag{1}$$

And the boundary between the ploughing and wedging regime is given by:

$$\theta = 0.5 \arccos f_{HK} \tag{2}$$

In these equations, θ is the attack angle of the sliding wedge and $f_{HK}=\tau/k$, where τ is the shear strength of the interface at the ploughing wedge and k is the shear strength of the soft plastically deforming counter surface. These equations are represented by the solid lines in Fig. 1. The dotted line is the exact boundary between the regimes which follows from [7]. It can be seen that the approximate relation is indeed very close to the exact solution. Next to ploughing wedges and spherical tips, ploughing cones [8] and pyramidal indenters [9,10] have also been analysed using the upper bound method. However, slipline models have some restrictions originating from the model assumptions. The most important restrictions are the neglect of elastic effects and the assumption of pure plastic material behaviour. So, at very small contact angles no ploughing is expected but elastic behaviour. In [16,17] limits due to elasticity have been analysed when indenting a plastically deforming substrate with a symmetric rigid wedge. If the criterion is applied to steel, elastic effects can be expected at attack angles lower than 18° for wedge shaped indenters. The criterion in fact represents the strain in the material due to indenting. When the results in the ploughing regime are compared with elastic–plastic FEM calculations, slipline solutions only give good results for attack angles higher than approximately 5° see [11]. The reason is that much higher strains than predicted by the slipline models develop close to the surface in the case of a ploughing wedge.

Secondly, failure of the ploughing tip itself can occur when sliding against a softer surface due to mechanical overloading despite its higher hardness. In [18,19] criteria are derived in terms of a critical tip angle for the sliding wedge and the hardness ratio between the sliding wedge and the softer flat.

Some of the basic assumptions like ideal plasticity of the soft surface and rigid behaviour of the hard asperity can be avoided using FEM models of ploughing asperities, one of the first being [11]. Later a single asperity moving over a countersurface has been simulated using meshless methods [12–15]. In these simulations, aspects like deformation of the ploughing asperity and material transfer to the ploughing asperity have been observed.

In this work, stability of pyramidal shapes will be investigated when ploughing through a plastically deforming material, using analytical models.

2. Modelling failure of asperities

2.1. Stress analysis for a simple triangular pyramid

Starting from a pyramidal asperity with a four sided base, only the front half will be in contact with the plastically deforming soft surface. The resulting geometry is a simple triangular pyramid loaded with forces due to the ploughing action of faces *BCD* and *ACD* as depicted in Fig. 2. If, only the front half of the four sided base supports the ploughing asperity, the asperity is supported by face *ABC*. Before discussing more complex situations, first this simple geometry will be analysed further.

The points *B* and *D* are respectively the extremes of the width *w* and the height *h*. The velocity vector of the moving soft counter surface relative to the fixed asperity is assumed to be acting in the negative *x*-direction. If a scratch is formed due to the ploughing action, behind the plane *ADB* no contact is expected. In reality the internal stress distribution is also dependent on the rear part of the asperity as will be discussed later. In the following, it will be assumed that the asperity is stationary and rigid with a plane of symmetry in the *xz*-plane. The asperity is loaded on face *BCD* (and because of the symmetry on face *ACD*) because of the ploughing action. Further, the rigid asperity is supported by face *OBC* (and because of the symmetry face *OAC*). In the analysis, the pressure p_{BCD} acting on face *BCD* will be called p_{pl}. p_{pl} is assumed to be constant over the whole contact area and directed inward normal to plane *BCD*. The tangential shear stress τ_{pl} is calculated using a Coulomb friction law, so $\tau_{pl} = \mu_{p_{pl}}$.

The geometry of the asperity is completely defined by *w*, *h* and the length *l* or in the dimensionless form, normalizing by *l*, two geometrical quantities remain

$$\bar{h} = \frac{h}{l} \tag{3}$$

$$\bar{w} = \frac{w}{l} \tag{4}$$

The coordinates of three points *B*, *D* and *C* are respectively given by (0, *w*, 0), (0, 0, *h*) and (*l*, 0, 0). These points describe a plane, which has the unit normal vector which will be denoted by n_{BCD}. The area of the triangle *BCD* is called A_{BDC}, and the area of triangle *ABC* is called A_{ABC} are given by

$$A_{BDC} = 1/2|BD \times BC| = 1/2\sqrt{(wh)^2 + (hl)^2 + (lw)^2} \tag{5}$$

$$\vec{n}_{BDC} = \frac{(wh, hl, lw)}{\sqrt{(wh)^2 + (hl)^2 + (lw)^2}} \tag{6}$$

$$A_{ABC} = wl \tag{7}$$

In the far field, the velocity vector is directed into the negative *x*-direction. The vector $\vec{t}$ is the tangent vector of *BDC* as close as possible to the direction of the plastic flow in the far field, so $\vec{x} \cdot \vec{t}$ has to be minimum. To minimize $\vec{x} \cdot \vec{t}$, vector $\vec{t}$ has to be

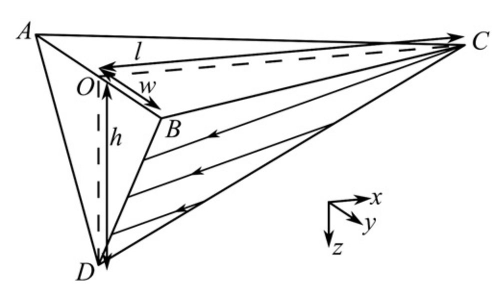

Fig. 2. Tip geometry with its dimension and flow lines on *BDC* of plastic deforming material.