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On estimations of maximum and average interfacial temperature rise in sliding elliptical contacts

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ABSTRACT

This study considers the approaches of Blok and Jaeger for the estimation of maximum and average temperatures in sliding elliptical contacts with both uniform and semi-ellipsoidal (Hertzian) heat distributions. The accuracy of each of these methods, which are based on single-point temperature matching between contacting bodies, is assessed relative to a numerical solution of the heat partition problem developed in a previous work, which imposed temperature matching at all nodal points. Comparisons are made for a wide range of Peclet numbers, as well as for moderate ranges of thermal conductivity ratio and elliptical ratio. It is found that the application of Blok's hypothesis yields remarkably accurate predictions of the maximum interfacial temperature, with typical errors less than 3%, whereas the hypothesis of Jaeger leads to predictions of the average interfacial temperature that have typical errors of around 6%. The authors also assess the accuracy of approximate formula developed by Tian and Kennedy to predict the maximum temperature at the interface for the case of sliding circular contacts and find the error to be no more than 2.6% for the full range of Peclet number. Further, the authors of the current study use fundamental heat source solutions developed by Tian and Kennedy to arrive at formulae for average temperature rise for circular contacts that are analogous to the Tian and Kennedy maximum temperature rise formulae. It is found that the formulae for computing the average interfacial temperature rise are also quite accurate, but have slightly more error than the maximum temperature rise formulae. Finally, in the present work, extensions are suggested to the maximum and average temperature rise formulae of Tian and Kennedy to include the effects of elliptical contact geometry. It is found that these formulae are at least 91% accurate for elliptical ratios between 0.4 and 5.0.

1. Introduction

Tribological and material behaviors of two bodies sliding against each other are greatly influenced by the temperature rise at the interface. This temperature rise depends on several factors such as load, sliding speed, frictional force, surface topography, thermal properties of the materials in contact, lubrication condition, and presence of other sources of heat transfer from the interface. Accurate prediction or estimation of the interface temperature rise is important for performing thermal stress analysis between two sliding bodies [1–3] and modeling thermal wear [1,2], both of which are relevant to many applications, such as machine tools [3,4], brake pads [5], gear teeth [6], and wheel–rail contacts [7]. In any case, a prediction of the maximum and/or average steady-state temperature rise at the interface of two sliding bodies can be valuable in designing against fatigue failure or other modes of system breakdown. For effective design of machine components, knowledge of the maximum and/or average interface temperature becomes important, as does the ease of being able to compute the same.

Blok [8] provided equations for computing maximum temperature rise due to circular uniform and paraboloidal (Hertzian contact pressure) heat distributions applied to a circular contact region, as well as for a uniform heat distribution applied to a square region. For computing interface temperature rise, Blok approximated the condition of continuity of temperature at the interface by equating the steady-state maximum temperature rise of both the bodies at the interface. That is, a total interfacial heating rate, attributed to frictional dissipation, is partitioned between the stationary and moving bodies so that the associated maximum temperature rises – as per the respective stationary and moving heat source models – are the same for each body. The notion that a good estimate for the maximum temperature rise can be found from Blok's approach to temperature matching will hereon be referred to as "Blok's hypothesis". Blok assumed that at low Peclet numbers the maximum steady state temperature is

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independent of the sliding speed and thus used the expression for the maximum temperature due to a stationary heat source to calculate the temperature rise at the interface. For high Peclet numbers, Blok suggested that the heat flow in the direction transverse to the sliding direction will be negligible and the square heat source can thus be approximated as an infinitely long band source. For intermediate Peclet numbers, Blok curve fitted the results of numerical integration to approximate the maximum temperature rise in the contact.

Jaeger [9] provided approximate equations for average temperature rise due to square and band shaped heat sources for very low Peclet numbers (<0.1) or for very high Peclet numbers (>10). For intermediate Peclet numbers Jaeger, like Blok, provided curve fit solutions for average and maximum temperature rise. Although band shaped contacts may be good approximations of several engineering contact regions, such as in meshing gear teeth and in a tool–chip interface, elliptical and circular contacts are more commonly seen in engineering applications.

Carslaw and Jaeger [10], in their classical book on heat conduction, provided equations for computing temperature rise due to heat sources with different shapes and heat distribution under both static and dynamic conditions.

Kuhlmann-Wilsdorf [11,12] modified the equation for maximum temperature rise put forth by Blok [13] to include the shape factor for elliptical contacts and used it to compute flash temperature due to friction and Joule heating. Kuhlmann-Wilsdorf used Jaeger's solution to derive a curve fit equation representing the dependence of maximum temperature on sliding velocity, and used the same reasoning to form approximate expressions for shape factor relating elliptical ratios with sliding velocity. However, Kuhlmann-Wilsdorf noted that the shape factor relation could not be accurately applied to elliptical geometry and hence limited her discussion to circular shaped contacts. Later on Kuhlmann-Wilsdorf [14,15] used those equations to determine flash temperatures in plastic contacts by accounting for changes in hardness due to flash temperatures.

Greenwood [16] put forth several interpolation formulae to calculate the maximum temperature rise in a body due to moving heat sources of circular, square and band shapes. Tian and Kennedy [17] presented solutions of temperature rise in a semi-infinite body due to uniform heat sources applied over circular and square regions and an ellipsoidal heat source applied over an elliptical region. In order to obtain the heat partition, they used Blok's approach of equating the maximum surface temperatures of both the bodies in the contact region.

Bansal and Streator [18] presented curve fit equations for computing maximum and average interface temperature in Hertzian contacts over a wide range of Peclet numbers, thermal conductivity ratios and elliptical ratios. These equations were based on fitting algebraic equations to numerical solutions of the heat partition problem, whereby the temperatures of the contact surfaces were matched at all nodal points [19]. A more comprehensive literature review on thermal analysis of interface temperature rise can be found in [19,20].

In the current study, we assess the accuracy of several means of estimating temperature rise of sliding contacts, including the hypotheses of Blok and Jaeger as well as the approach of Tian and Kennedy [17]. These methods represent much simpler approaches than that of Bansal and Streator [18] (and erratum [21]), in that they involve the matching of only one temperature value in the interface (i.e., either the maximum temperature or the average temperature), whereas, in Bansal and Streator there is temperature matching at all nodal points in the interface. It is of particular interest to assess the extent to which the simpler methods can be used with acceptable accuracy. It is noted here that the following application of the Blok hypothesis corrects that presented in a previous paper by the authors (see erratum [22]).

2. Interface temperature rise model

Bansal and Streator in [19] presented a least squares regression-based method for obtaining the steady-state temperature distribution at the interface of two sliding bodies, whose initial uniform temperatures may be the same or different. Both uniform and Hertzian contact pressure distributions were considered, which made the analysis applicable to both elastic and plastic contact pressures. Integral equations were developed expressing the temperature distribution of each body in terms of an unknown heat partition function. By assuming a polynomial form for the heat partition function and optimizing the coefficients to obtain the least squares difference in temperature at the interface between the two bodies (considering all nodal points in the interface), an estimate for the heat partition function was obtained.

For the sake of brevity, here only key equations are presented while readers are encouraged to visit [18,19] for a complete analysis. The definitions of the variables used here are same as those in [18]. The temperature distributions at the surface of two semi-infinite bodies sliding against each other with same initial temperatures (T_i) are given as [10]

$$T_1(x,y) = \frac{1}{2\pi k_1}\int\int \frac{q_1(x',y')}{\sqrt{(x-x')^2+(y-y')^2}}\,dx'\,dy' + T_i \tag{1}$$

$$T_2(x,y) = \frac{1}{2\pi k_2}\int\int q_2(x',y') \times \frac{\exp\left\{-(U/2\alpha_2)\left(\sqrt{(x-x')^2+(y-y')^2}-(x-x')\right)\right\}}{\sqrt{(x-x')^2+(y-y')^2}} \times dx'\,dy' + T_i \tag{2}$$

Here, Body 1 is stationary while the Body 2 is sliding to right with respect to Body 1, K_1 and K_2 are the thermal conductivities and α_1 and α_2 are thermal diffusivities of Body 1 and Body 2, respectively. Also, q_1 and q_2 are the heat flow rates per unit area into Bodies 1 and 2, respectively; let $q(x,y)$ be their sum or the total frictional heat generated at the interface, such that:

$$q(x,y) = q_1(x,y) + q_2(x,y) \tag{3}$$

The heat generation rate per unit area q at the interface due to friction can be expressed as:

$$\begin{aligned} q &= \mu p_m U && \text{for uniform contact pressure} \\ q &= \mu p_m U \frac{3}{2}\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}} && \text{for Hertzian contact pressure} \end{aligned} \tag{4}$$

where p_m is the average contact pressure over the contact area, U is the velocity of Body 2, a is the semi-axis in the sliding direction, b is the semi-axis in the transverse direction, and μ is the kinetic coefficient of friction.

Now let the heat partition factor, $\sigma(x,y)$, between the two bodies be defined as the ratio of the heat transfer into the moving body (Body 2) at (x,y) to the total heat generated at (x,y), i.e.:

$$\sigma(x,y) = \frac{q_2(x,y)}{q(x,y)} \tag{5}$$

Now let

$$q(x',y') = q_o f(x',y') \tag{6}$$