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Surface initiated tooth flank damage Part I: Numerical model

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ABSTRACT

A numerical model for the prediction of surface initiated damage on gear tooth flanks (micropitting and mild wear) is presented. This model hinges on a model of the mixed film lubrication regime and on the application of the Dang Van high-cycle multi-axial fatigue criterion. A simulation of the meshing of spur gears is performed in order to gain some understanding of the events associated with contact fatigue surface damage.

1. Introduction

The trend in gear design has ever been to improve the gear materials and surface treatments. This, alongside the improvement in lubricant oils formulation has allowed for ever higher speeds and power density in gear boxes. These improvements have helped in reducing the effect of the most destructive kinds of fatigue damage. In particular, the sort of progressive, in-depth originated fatigue damages, whose most characteristic example is the spalling damage, have been greatly reduced. As a result, surface originated fatigue damage have gained importance in the determination of the life of a gear [1,2].

These types of damage, and preponderant among them the micropitting damage, come from the propagation of cracks that initiate at the surface of the gear and progress, first inward and then outward, until a surface pit is produced [3]. A crucial characteristic of these fatigue cracks is that they have a very short propagation time [1]. Thus, in practice, the total life of a crack is almost equal to the initiation time. In this setting, an initiation model is very useful when dealing with such a contact fatigue mechanism.

Because these types of damage develop wholly within the first few tens of micrometers of the depth below the surface of a tooth, the important stress perturbations caused by the interaction of the roughness of the teeth cannot be ignored [4,5]. In practise, this translates itself in the need for the solution of the lubrication problem in the mixed film regime, in which important surface pressure distributions occur due to the interaction of the roughness peaks of each tooth with the surface of the opposing one, be that interaction through direct metal–metal contact, be it mediated by a very thin, highly pressurized lubricating film [6].

The aim of this work is to provide a numerical model for the prediction of the initiation of micropitting fatigue cracks and mild wear. This model hinges on the solution of the contact problem between gear teeth, characterized by the mixed film regime of elastohydrodinamic lubrication, and on the application of the Dang Van high-cycle multi-axial fatigue criterion, selected for its suitability for the prediction of fatigue crack initiation in complex loading cases.

As a final introductory remark, this work stems from Brandão's Master Thesis [7]: it is at once a summary of some of its most important findings and an extension of it.

2. Numerical model

The numerical model is based on the assumption, essentially true, that the roughness of a gear tooth flank is negligible in the transversal direction. This permits the use of the theory of the elastic half-space combined with plain strain, which greatly simplifies the calculations.

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Nomenclature

a	Hertzian half-width of the contact [m]
$\tilde{\tilde{A}}$	fourth order localization elastic tensor
C_s	specific heat of the surface material [J kg^{-1} K^{-1}]
F	contact load between the surfaces [N/m]
F^{BDR}	portion of the contact load borne by direct surface contact [N/m]
F^{EHD}	portion of the contact load borne by the lubricant film [N/m]
K	radius of the hyperspherical yield surface [Pa]
K_s	thermal conductivity of the surface material [W m^{-1} K^{-1}]
PT	thermal coefficient of both surfaces defined as $\mathrm{PT} = \sqrt{\rho_s C_s K_s}$
R_q	root mean square roughness parameter [m]
S_0	thermoviscosity parameter of the Roelands viscosity law
T_0	reference temperature [K]
T_f^{avg}	average lubricant temperature within the contact [K]
U_1	tangential velocity of the driving gear tooth surface [m/s]
U_2	tangential velocity of the driven gear tooth surface [m/s]
Z	piezoviscosity parameter of the Roelands viscosity law
f_Λ	load sharing function of the normal contact pressure between its lubricant film and direct contact borne portions
h_0	central film thickness [m]
n_1	driving gear rotational speed [rpm]
n_2	driven gear rotational speed [rpm]
$p^{BDR \cdot T}$	normal contact pressure when supposing that the surfaces are not lubrified and the entirety of the load is borne by direct surface contact [Pa]
p^{BDR}	portion of the normal contact pressure borne by direct surface contact [Pa]
$p^{EHD \cdot T}$	normal contact pressure when supposing that the surfaces are ideally smooth and the lubrication is in the full film EHD regime [Pa]
p^{EHD}	portion of the normal contact pressure borne by the lubricant film [Pa]
p^{MIX}	normal contact pressure (generally in the mixed film lubrication regime) [Pa]
p_H	hydrostatic stress of the mesoscopic stresses [Pa]

Greek letters

ΔT_f^{max}	greatest rise of the temperature of the lubricant above that of the surfaces [K]
ΔT_s^{max}	greatest rise of the temperature of the surfaces above that of the inlet [K]
Λ	specific lubricant film thickness [m]
$\tilde{\Sigma}$	mesoscopic stress tensor [Pa]
α_{DV}	Dang Van fatigue material properties
α_{τ_L}	oil's pressure dependence parameter of the limiting shear stress [Pa^{-1}]
β	thermoviscosity coefficient of the lubricant [K^{-1}]
β_{DV}	Dang Van fatigue material properties [Pa]
β_{eq}	equivalent reverse torsion stress [Pa]
β_{τ_L}	oil's temperature dependence parameter of the limiting shear stress [K]
$\dot{\gamma}$	in-plane shear strain rate [s^{-1}]
η	dynamic viscosity of the lubricant [Pa s]
η_0	dynamic viscosity at reference temperature T_0 and atmospheric pressure [Pa s]
μ	average friction coefficient within the contact
μ^{BDR}	average friction coefficient caused by boundary lubrication
μ^{EHD}	average friction coefficient caused by the lubricating film
$\tilde{\rho}$	stabilized mesoscopic residual stress tensor [Pa]
ρ_s	volumic mass of the surface material [kg m^{-3}]
$\tilde{\sigma}$	macroscopic stress tensor [Pa]
τ^{BDR}	portion of the tangential contact stress borne by direct surface contact [Pa]
τ^{EHD}	portion of the tangential contact stress borne by the lubricant film [Pa]
τ^{MIX}	tangential contact stress (generally in the mixed film lubrication regime) [Pa]
τ_L	limiting shear stress [Pa]
τ_{max}	maximum shear stress of the mesoscopic stresses [Pa]
τ_{L_0}	limiting shear stress at a reference temperature T_0 and atmospheric pressure [Pa]

As shown in Fig. 1, where a diagram of the numerical model is shown, the model may be divided into two main parts:

(1) The determination of the elastic stresses at each instant of the cycle.
(2) The application of the contact fatigue criterion.

In the first part of the model the geometry of the contact—which includes the roughness of the gear teeth flanks—and the contact force are determined, following which the contact loads between the driving and driven gear teeth are obtained through the application of a sub-model, to be later described in greater detail, which deals with mixed film lubrication. After that, it is a simple matter to calculate the stresses at every point under analysis. This procedure must be repeated for every instant of the loading cycle, in this case a meshing of gear teeth. When all the instants have been gone through, the entire history of the stresses induced by contact is known at each point.

In the second part of the model the residual stresses present before the cycle and the elastic stresses induced by the contact cycle are summed to form the macroscopic stresses (the term will be explained later). Based on these stresses, it is then possible to apply the contact fatigue criterion.

2.1. Mixed film lubrication model

2.1.1. Normal contact pressure

The contact between meshing gears is characterized by elastohydrodynamic (EHD) lubrication. In the classification of the several types of oil lubricated contact, the most relevant parameter is the specific lubricant film thickness (Λ):

$$\Lambda = \frac{h_0}{R_q} \tag{1}$$

where h_0 is the lubricant film thickness of the same contact but for an ideal smoothness of the surfaces and R_q is the composite RMS roughness of the contacting surfaces:

$$R_q = \sqrt{\frac{1}{\ell}\int_0^{\ell}(y_1 + y_2)^2 dx} \approx \sqrt{R_{q1}^2 + R_{q2}^2} \tag{2}$$