Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
Wear 262 (2007) 624–629
WEAR
www.elsevier.com/locate/wear

Resolving the contradiction of asperities plastic to elastic mode transition in current contact models of fractal rough surfaces

Yahav Morag, Izhak Etsion *

Department of Mechanical Engineering, Technion, Haifa 32000, Israel

Received 25 January 2006; received in revised form 20 June 2006; accepted 20 July 2006
Available online 8 September 2006

Abstract

The paper suggests a revision to the asperities plastic to elastic mode transition in the elastic–plastic contact model of fractal rough surfaces, offered by Majumdar and Bushan [A. Majumdar, B. Bushan (MB model) J. Tribol. 113 (1991) 1–11.]. According to the MB model, the contact mode of a single fractal asperity transfers from plastic to elastic when the load increases and the growing contact area exceeds a certain critical value, which is scale independent. This surprising result of the MB model is in contrast with classical contact mechanics where increasing contact area due to increased load is associated with a transition from elastic to plastic contact. The present study describes a revised elastic–plastic contact model of a single fractal asperity showing that, contrary to the MB model prediction, the critical contact area is scale dependent. The revised model also shows that a fractal asperity behaves as would be expected from classical contact mechanics namely, as the load and contact area increase a transition from elastic to plastic contact mode takes place in this order.

Keywords: Rough surface; Elastic–plastic contact; Fractal surface

1. Introduction

Contact mechanics of rough surfaces is important in studying and modeling physical phenomena such as thermal and electrical conductivity, friction, adhesion, wear, etc. Obviously, the ability to characterize surface profile by adequate parameters is crucial in these cases. Surface topography has been considered as a stationary random process [1], which can be characterized by statistical parameters such as the standard deviation of asperity heights σ, the slope σ', and the curvature σ'' [2]. However, modern roughness measurements by Sayles and Thomas [3] reveal that many engineered surfaces (mainly these used in MEMS) have a non-stationary surface texture, which has a multiscale nature. This means that when a section of a rough surface is magnified, smaller scales of roughness appear. Therefore, the parameters needed for stochastic models cannot be determined uniquely since they depend strongly on the resolution of the roughness-measuring instrument. The necessity for scale independent contact models motivated the growing use of fractal description of the multiscale nature of contacting rough surfaces. Archard [4] proposed in 1957 the first contact model, which used a "fractal" description. Although Archard's work predates the use of the term *fractal*, he recognized this general characteristic of surfaces, and suggested a model of rough surfaces in which a progression of smaller hemispherical asperities were superposed on a larger scale. He showed that even for a purely elastic contact, a linear relationship between the load and the contact area can be established, when a Hertzian contact is assumed. Ciavarella and Demelio [5] revisited the Archard's model for an elastic multiscale contact of rough surfaces, and compared it with modern fractal models. Berry and Lewis [6] investigated the properties of the Weierstrass and Mandelbrot fractal function (WM), which is a discrete series of superposed self-affine sine waves, and forms the basis for fractal surface roughness description. Majumdar and Bushan [7], and Majumdar and Tien [8], used this function to define a simple idealization of a two-dimensional fractal rough surface profile. Greenwood and Wu [9] criticized this approach, claiming that the results presented in Ref. [7,8] were based primarily on a model of a continuous (and not discrete) power-law spectral density.

Majumdar and Bushan [10], used the WM function to develop one of the first fractal contact models, referred to in the following

* Corresponding author. Tel.: +972 4 829 2096; fax: +972 4 829 5711.
E-mail address: etsion@tx.technion.ac.il (I. Etsion).

0043-1648/$ – see front matter © 2006 Elsevier B.V. All rights reserved.
doi:10.1016/j.wear.2006.07.007

as the MB model. The rough surface texture in the MB model is based on a 2D multiscale surface profile $z(x)$ generated by the WM function, in which the surface roughness is given by

$$z(x) = G^{D-1} \sum_{n=n_1}^{\infty} \frac{\cos(2\pi\gamma^n x)}{\gamma^{(2-D)n}} \qquad (1)$$

where D is the fractal dimension of the surface profile (for a physically continuous surface $1 < D < 2$), and γ^n determines the frequency spectrum of the surface roughness ($\gamma > 1$). The fractal roughness parameter, G, is a characteristic length scale of the surface that determines the position of the spectrum along the power axis, and is invariant with respect to all frequencies of roughness. The index, n, indicates the frequency level of the asperities. As can be seen from Eq. (1) the WM function is an infinite series of cosinusoidal waves of different amplitudes and frequencies, superimposed on each other. The frequency difference between the levels leads to anywhere between constructive to destructive interference. The frequency ratio between two adjacent levels is $\gamma^{n+1}/\gamma^n = \gamma$, and the amplitude ratio is $1/\gamma^{(2-D)}$. Since $\gamma = 1.5$ was found to be a suitable value for high spectral density and for phase randomization [8], the number of wavelengths in a certain level, is 1.5 times the number of wavelengths in the previous level.

The MB model assumes that the size distribution of contact area spots follows that of ocean islands generated by truncating the earth surface at a constant height. This model triggered rough surface contact studies by different researchers, e.g. Zahouani et al. [11], and Willner [12], and its basic ideas were used in various fields of applied physics. Komvopoulos and Yan [13] developed an algorithm to generate a three-dimensional fractal surface using the WM function and incorporated it into an elastic–plastic contact model. Ciavarella et al. [14] used a fractal model concept to investigate elastic contact stiffness and contact resistance. Bora et al. [15] developed a method to investigate the geometry of asperities of Silicon MEMS surfaces at different length scales. Sahoo and Chowdhury [16,17] analyzed fractal friction and wear. Kogut and Komvopoulos [18,19] applied the fractal surface concept to the new emerging field of contact electro-mechanics, and Kogut and Jackson [20], very recently, compared the statistical and fractal approaches to contact modeling, showing substantial differences between the two.

As we will show in the next section, the MB model suffers from a drawback in treating the transition from elastic to plastic contact mode of a fractal surface single asperity. This drawback may affect many areas of tribology such as contact conductance, wear, adhesion, friction, MEMS interfaces, etc., where the MB model concept has been used or referred to e.g. [14–31]. The main objective of this paper is therefore to resolve the above-mentioned drawback by offering a revision of that aspect in the MB model.

2. Description of the MB model

Fig. 1 illustrates the conceptual approach of the MB model. Fig. 1(a) presents the idea of Greenwood & Tripp [32], of replacing two contacting rough surfaces, separated by a distance d, with an equivalent rough surface in contact with a rigid flat. The equivalent rough surface is generated by the WM function (see Eq. (1)), and the geometry of each asperity (Fig. 1(b)) is represented by the appropriate single term in the cosine series (an assumption that Greenwood and Wu [9] find difficult to accept). The 2D asperity profile $z(x)$ has, in the MB model, the form:

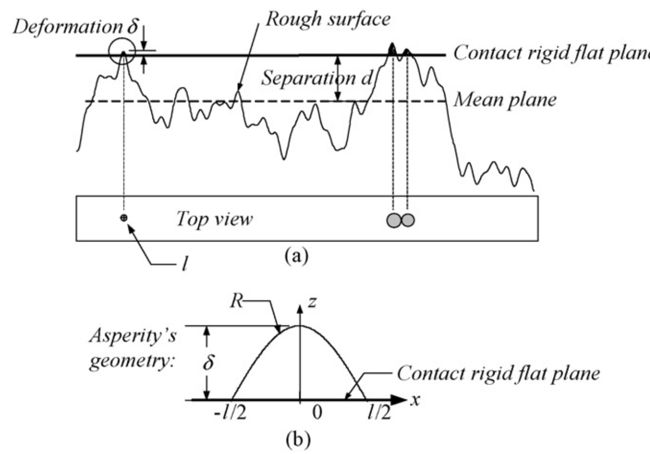

Fig. 1. The MB model: (a) contact between a rough surface and a flat producing isolated contact spots, and (b) the geometry of a contact spot of length scale l.

$$z(x) = G^{D-1} l^{2-D} \cos\left(\frac{\pi x}{l}\right) \qquad (2)$$

where l is the length scale (base diameter) of a fractal asperity at level n, such that $l_n = 1/\gamma^n$. Similar to Ref. [9], in our opinion Eq. (2) should contain additional factor 2 in the parenthesis (see Eq. (1)), but we choose to continue with a quote of the original expression shown in the MB model. Following Eq. (2), the asperity height is

$$\delta = z(0) = G^{D-1} l^{2-D} \qquad (3)$$

In addition, the radius of curvature at the asperity summit is:

$$R = \frac{1}{|\mathrm{d}^2 z/\mathrm{d}x^2|_{x=0}} = \frac{l^D}{\pi^2 G^{D-1}} \qquad (4)$$

It can be seen from Eq. (4) that the asperity radius of curvature, depends on its length scale, contrary to the classical model of Greenwood and Williamson [2] (the GW model), which assumes constant radius for all the asperities.

In the MB model, it is assumed that when the two surfaces are brought into contact the rigid flat forms a "truncation plane" on which the contact spots are spread. The size distribution of the contact spots follows the model of Mandelbrot [33], i.e. obeys a fractal law of the distribution of island areas truncated by the sea level. According to the MB model the truncation area, a_t, of a single asperity forms its base as well as its real contact area, a, where $a = l^2$ (π was omitted in Ref. [10]). Hence, by definition, in the MB model the asperity is fully deformed by the contacting plane, and so the interference of any specific asperity is identical to its full height δ. By substituting $l = a^{1/2}$ in Eqs. (3) and (4), Majumdar and Bushan obtained the asperity height δ, and its radius of curvature R, in terms of the truncated contact area a:

$$\delta = G^{D-1} a^{(2-D)/2} \qquad (5)$$