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Time–frequency analysis of friction-induced vibration under reciprocating sliding conditions

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Abstract

A time–frequency analysis can give an overall view of the behaviour of friction-induced vibration. In this paper, short-time Fourier transform (STFT), Wigner–Ville distribution (WVD), Choi–Williams distribution (CWD) and Zhao–Atlas–Marks distribution (ZAMD) are applied to analyze time–frequency characteristics of friction-induced vibration. The result shows that there is always a frequency change in the time–frequency presentation of vibration in the location where the vibration is bounded. The frequency changes in time–frequency presentations are associated with nonlinearity of vibration systems. The nonlinearity may be counted as the evidence to support the consideration that friction-induced vibrations are bounded by limit cycles due to the system nonlinearity. Based on the time–frequency presentations of vibrations, it may be concluded that the friction vibration system is generally a linear system in the phase of vibration initiation but is a nonlinear system in the phases of vibration being bounded and disappearance.

Keywords: Time–frequency analysis; Friction-induced vibration; Squeal; Reciprocating sliding

1. Introduction

Phenomena concerning squeal are very confusing. Two friction sliding processes in nominally identical conditions may show very different propensities of generation of squeal. Further, without changing the nominal conditions, a sliding may become silent from squealing or vice versa [1,2]. Many endeavors have been made to acquire a comprehensive understanding of squeal [3–9]. Four excitation mechanisms of squeal are proposed in the literature. These are stick–slip, negative friction–velocity slope, sprag slip and modal coupling [3–6]. During the 1980s and 1990s, the emphasis of studying squeal was shifted from stick–slip to the modal coupling between normal and friction forces. Until recently, the modal coupling is still considered as a major mechanism behind squeal generation [10–14]. It has been being the main topic of many researches such as a finite element analysis and a dynamics simulation concerning squeal and associated vibration [10–13]. The modal coupling generally involves an eigenvalue analysis of friction system motion. However, it is seen that such an eigenvalue analysis cannot define clearly physical phenomena causing squeal [9–13]. According to Tworzydlo, it appears that there is a mechanism different from the above-mentioned four mechanisms for the generation of friction-induced vibration [10]. Tworzydlo observed that one vibration occurred accompanying the development of surface damage of the wear scar. In view of the theories of negative friction–velocity slope and modal coupling, the vibration will grow infinitely once the friction system becomes unstable. Actually, this vibration is always limited in magnitude. In the field of friction-induced vibration, a common explanation for the limited magnitude of friction-induced vibration is that it may be bounded by a limit cycle due to the system inherent nonlinearity [3]. However, this nonlinearity has not been significantly pursued in the disc brake squeal literature [14]. In a word, there is a strong need for further research to promote our understanding of various friction mechanisms behind squeal generation.

Squeal is very closely associated with friction-induced vibration. It is commonly considered that vibration emits squeal [3].

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Therefore, the study on squeal is generally shifted to that on friction-induced vibration. The whole process from the initial formation to disappearance of friction-induced vibration may contain abundant information about the mechanism of the vibration formation. Time–frequency analysis is a good consideration for this process analysis. The time–frequency presentation is suited for analyses of stationary or non-stationary signals. It gives a time and frequency domain representation of the signal simultaneously. Time–frequency representation has been applied to many fields including assessment of physical condition of mechanical systems [15], tribology [16,17] and structural vibration analysis [18].

In this paper, the emphasis is given to a time–frequency analysis of the vibration signal associated with squeal. Time–frequency representations of a whole process from the initial formation to disappearance of friction-induced vibration are investigated. In the present analysis, four methods including short-time Fourier transform (STFT), Wigner–Ville distribution (WVD), Choi–Williams distribution (CWD) and Zhao–Atlas–Marks distribution (ZAMD) are considered to obtain better accuracy of the time–frequency analysis. Based on a comparative study of these four methods, STFT is adopted as a rough method and ZAMD is adopted as a fine method to analyze time–frequency representations of vibration signals.

2. Theoretical background of time–frequency analysis

2.1. Short-time Fourier transform (STFT)

The short-time Fourier transform divides up a signal into small time segments and performs Fourier transforms on each segment of time to derive the spectra. The short time–frequency Fourier transform of a signal $x(t)$ can be expressed by,

$$w(t, \omega) = \frac{1}{\sqrt{2\pi}} \int e^{-j\omega\tau} x(\tau) h(\tau - t)\, d\tau \tag{1}$$

where $h(t)$ is a window function centered at time t. The energy density spectrum of the short-time Fourier transform is defined as,

$$P = |w(t, \omega)|^2 = \left| \frac{1}{\sqrt{2\pi}} \int e^{-j\omega\tau} x(\tau) h(\tau - t)\, d\tau \right|^2 \tag{2}$$

STFT approach can successfully deal with slowly varying signals, but cannot process properly signals consisting of many harmonics or presenting resonance phenomena. Its resolution in time and frequency domains is heavily dependent on applied windows. Although the STFT compromise between time and frequency information can be useful, the drawback is that once you choose a particular size for the time window, that window is the same for all frequencies. More significantly, this method shows a clear advantage that there is no cross-terms interference in its spectrum analysis results, which is annoying in the spectrum analysis results from Wigner–Ville distribution, Choi–Williams distribution and Zhao–Atlas–Marks distribution.

2.2. Wigner–Ville distribution (WVD)

Wigner–Ville distribution function introduced by Wigner [19] is as follows,

$$w(t, \omega) = \int e^{-j\omega\tau} x\left(\frac{t+\tau}{2}\right) x^*\left(\frac{t-\tau}{2}\right) d\tau \tag{3}$$

where $x^*(t)$ is the complex conjugate of $x(t)$.

WVD is more attractive because it discards the hypothesis of short-term stationarity of the signal and overcome the typical problem of the compromise between time and frequency resolution. However, this method shows a clear drawback consisting of the appearance in the spectrum of artifacts called 'cross-terms'. Sometimes, the interference from the cross-terms may result in a misidentification of the vibration frequencies.

2.3. Choi–Williams distribution (CWD)

A generalized class of time–frequency presentations was introduced by Cohen [20] as follows,

$$w(t, \omega) = \frac{1}{2\pi} \iiint e^{-j\theta t - j\omega\tau + j\theta\omega} \phi(\theta, \tau) x \times \left(\frac{u+\tau}{2}\right) x^*\left(\frac{u-\tau}{2}\right) du\, d\tau\, d\theta \tag{4}$$

where $x(u)$ is the signal needed to analyze, $x^*(t)$ its complex conjugate and $\phi(\theta, \tau)$ is an arbitrary function called the kernel. By choosing different kernels, different distributions are obtained. When kernel $\phi(\theta, \tau) = 1$, Eq. (4) is well-known WVD. Choi–Williams introduced the exponential kernel of the following form,

$$\phi(\theta, \tau) = e^{-\theta^2\tau^2/\sigma} \tag{5}$$

where σ is a parameter. If σ is large enough then the kernel approaches 1, Choi–Williams distribution approaches the Wigner–Ville distribution. For a small σ, it peaks at the origin and falls off rapidly away from the axis. This property contributes to reducing the cross-term in the case of multi-component signals. The power spectrum density of Choi–Williams distribution is expressed by,

$$w(t, \omega) = \frac{1}{4\pi^{3/2}} \iint \frac{1}{\sqrt{\tau^2/\sigma}} e^{-((u-t)^2/(4\tau^2/\sigma)) - j\omega\tau} x \times \left(\frac{u+\tau}{2}\right) x^*\left(\frac{u-\tau}{2}\right) du\, d\tau \tag{6}$$

2.4. Zhao–Atlas–Marks distribution (ZAMD)

In the ZAMD method [21], the kernel is adopted as follows,

$$\phi(\theta, \tau) = g(\tau)|\tau| \frac{\sin(a\theta\tau)}{a\theta\tau} \tag{7}$$