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Wear 260 (2006) 567–572
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Short communication

On the application of Dang Van criterion to rolling contact fatigue

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Received 24 August 2004; received in revised form 18 February 2005; accepted 8 March 2005
Available online 15 April 2005

Abstract

In this note, the problem of the calibration of the Dang Van multiaxial fatigue criterion is addressed. The discussion is based on uniaxial fatigue tests performed with different stress ratios. Results show that the usual technique for calibrating the constants of the Dang Van criterion does not agree with experimental evidence, especially for negative stress ratios. For this reason, a different fatigue failure locus made of two straight line segments is proposed and typical three-dimensional rolling contact stress histories are analyzed using the traditional and proposed methods. Results show that the conventional technique does not agree with knowledge coming from shakedown approaches of rolling contact while the proposed method seems to constitute a more appropriate limit.

Keywords: Rolling contact fatigue; Multiaxial fatigue; Shakedown; Residual stresses; Stress ratio

1. Introduction

High cycle contact fatigue failure has considerable industrial relevance for those applications where contact loads appear, as for example, gears, cams, rolling bearings and rail–wheel systems. In particular, rolling contact fatigue (RCF) is maybe one of the most difficult problems regarding out-of-phase multiaxial fatigue because all six components of the stress tensor may arise. In recent years, a high number of papers have been dealing with the use of the Dang Van multiaxial criterion for RCF (see, for example, references [1–11]). The aim of this note is to discuss the suitability of this method to RCF, by verifying its response to several uniaxial tests and extrapolating data to three-dimensional contact stress histories.

1.1. Dang Van's fatigue criterion

Before detailing the calibration of the Dang Van locus, an outline of the practical application of this criterion will be given. The basis of the following relationships is the application of the elastic shakedown principles at the mesoscopic scale, which will be shortly explained in this article. For more theoretical details, the interested reader is referred to [12,13]. The Dang Van criterion can be expressed by:

$$\tau_{\max}(t) + a_{\mathrm{DV}}\sigma_{\mathrm{H}}(t) = \tau_{\mathrm{W}} \quad (1)$$

with a_{DV} being a constant to be determined, τ_{W} the fatigue limit in reversed torsion, $\sigma_{\mathrm{H}}(t)$ the instantaneous hydrostatic component of the stress tensor and $\tau_{\max}(t)$ the instantaneous value of the Tresca shear stress, i.e.,

$$\tau_{\max}(t) = \frac{\hat{s}_{\mathrm{I}}(t) - \hat{s}_{\mathrm{III}}(t)}{2} \quad (2)$$

evaluated over a symmetrized stress deviator, which is obtained by subtracting from the stress deviator:

$$s_{ij}(t) = \sigma_{ij}(t) - \delta_{ij}\sigma_{\mathrm{H}}(t) \quad (3)$$

a constant tensor, $s_{ij,\mathrm{m}}$, i.e.,

$$\hat{s}_{ij}(t) = s_{ij}(t) - s_{ij,\mathrm{m}} \quad (4)$$

The constant tensor $s_{ij,\mathrm{m}}$ is defined by the relationship:

$$\max_t[(s_{ij}(t) - s_{ij,m})(s_{ij}(t) - s_{ij,m})] = \min_{s'_{ij}} \max_t[(s_{ij}(t) - s'_{ij})(s_{ij}(t) - s'_{ij})] \quad (5)$$

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doi:10.1016/j.wear.2005.03.007

The constant tensor $s_{ij,\mathrm{m}}$ may be regarded as the part of the stress deviator, which has no influence on the fatigue crack nucleation, and therefore, is eliminated through the minimization process of Eq. (5). One of the consequences of this method is the correct prediction of the absence of any effect of a mean shear stress upon the torsional fatigue limit.

In Dang Van's original proposal, the existence of the constant stress deviator $s_{ij,m}$ is justified by the assumption that the stress deviator defined by Eq. (4) is the mesoscopic stress deviator, i.e., the stress state found at the grain scale. Close to the fatigue limit, some unfavorably oriented grains may still undergo cyclic plasticity, although the macroscopic behaviour appears elastic. For crack nucleation to be avoided, these grains necessarily have to reach an elastic shakedown state. The presence of the residual stress deviator defined by Eq. (5) allows fulfillment of this condition.

Defined at the mesoscopic scale, these residual stresses are different from those developing as a consequence of a structural elastic shakedown. In this case, the minimization procedure of Eq. (5) would not be enough and the evaluation of residual stresses would require the classical procedures of the theory of plasticity.

If the macroscopic stress state exceeded the yield limit, because of the assumption of the elastic behaviour of the crystalline aggregate surrounding the unfavorably oriented grains, the Dang Van criterion would not be applicable, unless the material shakes down to the elastic state also at the macroscopic level. For this reason, Dang Van criterion can be applied to some rolling contact fatigue problems, provided that the contact conditions allow for the elastic shakedown of the material subjected to contact stresses.

Going back to Eq. (1) because of the symmetrization of the stress deviator, the term $\tau_{\max}(t)$ alone cannot account for the effect of normal stresses upon the fatigue limit. For this reason, the effect of a mean normal stress upon fatigue limits in bending and torsion is taken into account by introducing into Eq. (1) the term $\alpha_{\mathrm{DV}}\sigma_{\mathrm{H}}(t)$. This term represents the effect of the hydrostatic stress on crack nucleation and it can also be demonstrated that macroscopic and mesoscopic hydrostatic stresses are the same [12].

If residual stresses are superimposed on the applied stresses, the term $\tau_{\max}(t)$ is not altered because of the minimization process of Eq. (5). However, the term $\alpha_{\mathrm{DV}}\sigma_{\mathrm{H}}(t)$ is modified by the presence of residual stresses through the hydrostatic part of the residual stress tensor $\sigma_{\mathrm{H,res}}$, and therefore, Eq. (1) becomes:

$$\tau_{\max}(t) + a_{\mathrm{DV}}[\sigma_{\mathrm{H}}(t) + \sigma_{\mathrm{H,res}}] = \tau_{\mathrm{W}} \quad (6)$$

The constant a_{DV} appearing in the expression of the Dang Van criterion is usually calibrated with two fatigue tests, pure bending (or tension–compression) and pure torsion [1]. In the space constituted by the symmetrized shear ($\tau_{\max}$) and the hydrostatic stress (σ_{H}), reversed torsion and alternating bending are represented by a vertical line segment and a V-shape curve, respectively. As shown schematically in Fig. 1,

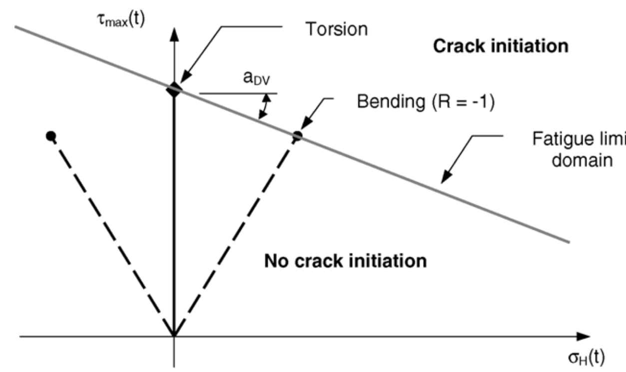

Fig. 1. The Dang Van diagram: a scheme of typical calibration with bending and torsion fatigue test.

the fatigue locus is then assumed as the straight line tangent to these two lines, with a constant slope given by the following expression:

$$a_{\mathrm{DV}} = 3\left(\frac{\tau_{\mathrm{W}}}{\sigma_{\mathrm{W}}} - \frac{1}{2}\right) \quad (7)$$

Peridas and Hills [14] have recently pointed out the importance of using more than these two tests. The first aim of their proposal is to obtain a more accurate fatigue limit domain. In fact, as the limit line is tangent to the paths corresponding to pure bending and torsion tests, which are close to each other, small errors may have a profound effect on the slope of the locus line. The second and maybe more important aim of their proposal is to confirm, on the basis of a more extended amount of data coming from experiments, the suitability of the Dang Van criterion to reproduce simple experimental cases.

On this context, in this work, the Dang Van criterion is tested with a new data set coming from uniaxial experiments performed at different stress ratios and the results are interpreted in terms of fatigue failure predictions for typical rolling contact stress histories.

2. Calibration of Dang Van fatigue failure locus with smooth specimens

In this section, the calibration of Dang Van fatigue failure locus will be performed using smooth specimens. In a previous work [15], bending and axial tests were performed at different stress ratios, either on smooth and notched specimens. The material was a quenched and tempered steel with an ultimate tensile strength of 1350 MPa, and 0.6% elongation at fracture. In this work, we will focus only on the results obtained with smooth specimens. Table 1 summarises these results. It can be observed that the values of the stress am-

Table 1
Summary of normalised (to $\sigma_{a,R=-1}$) fatigue limit $\sigma_a/\sigma_{a,R=-1}$

Stress ratio (R)	−2	−1	0.1	0.3
Normalised fatigue limit	1	1	0.76	0.68