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A quantitative geometric mechanics lens model: Insights into the mechanisms of accommodation and presbyopia

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ABSTRACT

This study expands on a geometric model of ocular accommodation (Reilly and Ravi, Vision Res. 50:330–336; 2010) by relaxing assumptions regarding lens symmetry about the equator. A method for predicting stretching force was derived. Two models were then developed: Model 1 held the equatorial geometry constant at all stages of accommodation, while Model 2 allowed localized deformation at the equator. Both models were compared to recent data for axial thickness, anterior and posterior radii of curvature, surface area, cross-sectional area, volume, and stretching force for the 29-year-old lens. Age-related changes in accommodation were also simulated. Model 1 gave predictions which agreed with the Helmholtz theory of accommodation, while Model 2's predictions agreed with the Schachar mechanism of accommodation. Trends predicted by Model 1 agreed with all available experimental data, while Model 2 disagreed with recent surface area measurements. Further analysis indicated that Model 1 was fundamentally more efficient in that it required less force per diopter change in optical power than Model 2. Model 1 more accurately predicted age-related changes in accommodation amplitude. This implies that the zero-force (fully accommodated) state geometry changes with age due to a shifting balance in residual stresses between the lens and capsule.

1. Introduction

Accommodation is the process by which the eye dynamically changes its focal length by modulating the force applied to the lens. Presbyopia is the continuous decline in accommodation ability with age which occurs in all humans and primates (Petrash, 2013). The von Helmholtz (1855) theory of accommodation states that the ciliary muscle applies a tensile force on the zonules to decrease the optical power of the lens, thereby allowing clear vision at distance. Gullstrand (1924) later pointed out that the lens capsule is pivotal in shaping the lens substance in the process of accommodation by distributing the zonular load across the surface area of the lens fiber cells. Recently, Schachar, Black, Kash, Cudmore, and Schanzlin (1995) proposed a modification of the Tscherning (1904) alternative hypothesis in which the equatorial radius and stretching force are maximized at maximum accommodation.

Presbyopia is the progressive decline of accommodation with age. Despite extensive study, the driving force for presbyopia remains unclear (Weale, 1999). It is most commonly held that an increase in lens stiffness diminishes the lens' ability to change its shape during accommodation (Glasser and Campbell, 1998; Heys, Cram and Truscott, 2004; Weeber et al., 2007; Weeber and van der Heijde, 2007). Numerous treatments for presbyopia have been developed; unfortunately, none of these approaches offers all of the benefits of the natural mechanism of accommodation. Therefore, efforts have been made to replace the aged lens with a biomimetic substitute matching the behavior of the natural lens using a process called "lens refilling" (Kessler, 1964; Koopmans et al., 2003; Koopmans et al., 2006; Koopmans et al., 2004; Parel et al., 1986; Reilly et al., 2009).

The success of lens refilling depends on the development of a robust understanding of the optomechanical mechanism of accommodation and its age-related decline so that the prosthesis might mimic the behavior of the young, healthy, natural lens. A number of increasingly complex mechanical models attempt to explain this process (Burd, Judge and Cross, 2002; Burd, Judge and Flavell, 1999; Chien, Huang and Schachar, 2006; Hermans et al., 2006; Koretz and Handelman, 1982, 1983; Koretz, Handelman and Brown, 1984; O'Neill and Doyle, 1968; Schachar, Huang and Huang, 1993; Weeber and van der Heijde, 2007). The accuracy of such models depends on the accuracy of the input data, namely: the geometry (Chien, Huang and Schachar, 2003; Kasprzak, 2000; Kong et al., 2009) and mechanical properties of the lens (Fisher, 1971; Heys, Cram & Truscott, 2004; Reilly & Ravi, 2009; Weeber et al., 2007) and its capsule (David and Humphrey, 2007; David

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et al., 2007; Fisher, 1969a; Krag and Andreassen, 2003; Pedrigi, David et al., 2007; Pedrigi et al., 2009; Pedrigi, Staff et al., 2007). The quality of mechanical property data for both the lens and its capsule have been called into question (Burd, 2009; Burd, Wilde & Judge, 2006; Schachar, 2007), implicitly casting doubt on predictions of the models which depend on these values.

In light of these difficulties, a geometric model which described the accommodative mechanism in the young lens without involving the mechanical properties of the lens itself was previously developed (Reilly & Ravi, 2010). Normalized changes in the lens' optomechanical parameters were found to change in a manner qualitatively similar to that predicted by the Helmholtz theory of accommodation upon equatorial loading for a variety of lens-shaped capsules surrounding an incompressible material.

In the present study, this model is refined by incorporating asymmetry about the equatorial plane and used recent data to more accurately estimate the disaccommodated geometry of the lens. A slight variation was also derived which predicts accommodation via the mechanism postulated by Schachar et al. (1995) in which localized deformation at the equator yields an increase in optical power upon the application of equatorial tension. The predictions for lens behavior as a function of age and accommodation are quantitatively compared to the most recent available data. The results offer insight into the consistency of published geometric and mechanical metrics for lens performance and give new insights into the driving force for presbyopia.

2. Methods

The principal methods used to predict changes in lens optical parameters due to an increase in equatorial diameter have been described previously (Reilly & Ravi, 2010). Briefly, this method predicts changes in optical (axial thickness t, anterior and posterior central radii of curvature R_A and R_P, and central optical power P) and mechanical (surface area S, cross-sectional area C, volume V, and stretching force F) parameters due to changes in equatorial radius a. The model makes two key assumptions: the geometric shape class of the lens does not change between accommodation and disaccommodation, and that the lens' volume is unaffected by accommodative state (isochoricity). A schematic lens is shown in Fig. 1.

The present study attempts to generate quantitative predictions of the optical and mechanical responses by removing some of the simplifying assumptions used previously. The lens was represented by a more general formulation in which the anterior and posterior segments of the lens may be represented by different geometries rather than assuming symmetry about the equatorial plane. Specifically, by forcing the volume of each segment to be held constant during accommodation, the solutions found by Reilly and Ravi (2010) still hold. This assumption was based on the findings of Hermans et al. (2009), who noted that the relative segment thicknesses varied by less than 4% from the fully disaccommodated to the fully accommodated state, and that the lens' volume was independent of accommodative state. Based on this result, the anterior:posterior segment volume ratio was held fixed as 0.7. All calculations were carried out using MATLAB 2013b (The Mathworks, Inc.; Natick, MA). Since the present lens model is asymmetric about the equatorial plane, the general thick lens formula was used to compute the optical power P:

$$P = (n_\ell - n_A)\left(\frac{1}{R_A} + \frac{1}{R_P} - \frac{t(n_\ell - n_A)}{n_\ell R_A R_P}\right), \tag{1}$$

where n_A was the refractive index of aqueous and vitreous humors (1.335; Swindle, Hamilton & Ravi, 2008) and n_ℓ was the age-dependent equivalent refractive index of the lens as determined by Dubbelman, van der Heijde and Weeber (2005b).

2.1. Specification of the initial lens geometry

A preliminary investigation indicated that the torispherical dome was capable of giving better quantitative predictions than any of the other geometric shape classes described in Reilly and Ravi (2010). Therefore, the lens was described as a torispherical dome with an initial shape based on the fully disaccommodated data from Dubbelman, van der Heijde and Weeber (2001, 2005a) and Strenk et al. (1999) (Table 1). The torispherical dome's cross-sectional profile $z(r)$ is given by

$$z(r) = \begin{cases} t - R + \sqrt{R^2 - r^2} & \text{if } r \leqslant r_c \\ \sqrt{R_t^2 - (c - r)^2} & \text{if } r \geqslant r_c, \end{cases} \tag{2}$$

where R_t is the radius of the torus (also called the knuckle radius), c is the distance from the optical axis to the center of the torus (i.e. $c + R_t = a$), and r_c is the critical radius at which the torus and spherical cap intersect. The critical radius r_c is given by

$$r_c = c\left[1 + \left(\frac{R}{R_t} - 1\right)^{-1}\right]. \tag{3}$$

The previously-described method allowed for arbitrary prescription of the knuckle radius R_t. Here, this value was computed from data describing the fully disaccommodated lens geometry ($R_{t,0}$) as

$$R_{t,0} = \frac{b^2 + a^2 - 2Rb}{2(a - R)}, \tag{4}$$

where b is the thickness of the segment (i.e. b_a is the thickness of the anterior segment of the lens and b_p is the thickness of the posterior segment such that $t = b_a + b_p$). Note that R_t was generally different for the anterior and posterior segments and was computed based on the anterior or posterior value of R.

2.2. Simulation of accommodation

Once the initial state of the lens was described in this manner, the process of accommodation was simulated by systematically changing the equatorial radius of the lens according to the stretch ratio λ, which is related to the equatorial radius a by

$$\lambda = \frac{a}{a_0}, \tag{5}$$

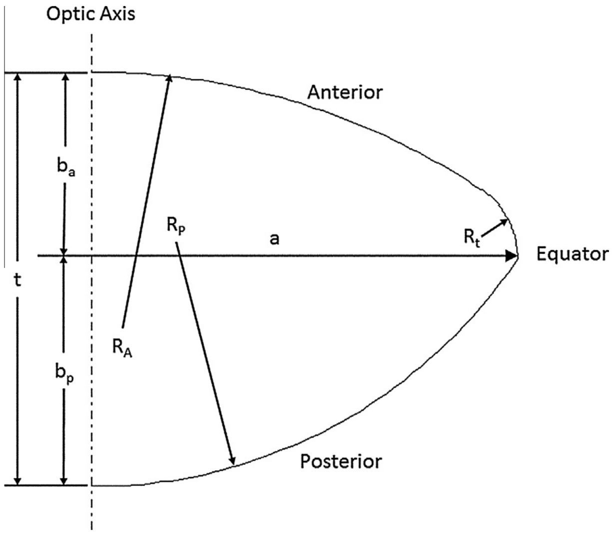

Fig. 1. Schematic of the torispherical dome geometric model for the human lens.