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Combining two orthogonal secondary flows to enhance the mixing in an annular duct

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ABSTRACT

Laminar flows in annular tubes can be encountered in industrial situations. In order to improve static and continuous mixers or multifunctional heat exchangers, several ways have been investigated. In the present article, two already well-known kinds of passive enhancement devices are set on a laminar annular flow and numerically studied. Alternate elliptic wall deformations are applied to the external wall as the internal wall is swirled. These geometrical perturbations both create secondary flows that are in mutually orthogonal directions. Their combination is numerically evaluated in this paper and is found to lead to a significant increase of the mixing. A nearly uniform mixing is even achieved for a reduced tube length as shown by considering scalar mixing for different inlet configurations. The analysis, based on Lagrangian tracking and Poincaré sections, shows that the mixing enhancement can be explained by the formation of zones of chaotic advection in the flow.

Keywords: Mixing enhancement; Tube wall deformation; Swirled flow; Numerical simulations; Chaotic advection

1. Introduction

The enhancement of mixing in laminar regime is a crucial issue in several industrial fields. Laminar flows are indeed needed in many situations such as those that involve very viscous fluids, for example in the food industry, or such as micro-scale devices (Xia et al., 2012; Zheng et al., 2013a) and more generally in the process industry. Moreover, numerous applications need mixing intensification. One can cite mixers for the chemical process industry (Thakur et al., 2003; Meijer et al., 2012), devices that use the increase of mixing to enhance the heat transfer (Mokrani et al., 1997; Jiang et al., 2004) or multifunctional heat exchangers. This latest recent type of device has widely been studied as it constitutes a way to enhance chemical reaction rates by controlling the heat transfer (Anxionnaz et al., 2008). The process costs and safety are thus improved by allowing the transition from batch systems to continuous systems.

In order to increase mixing, a large number of solutions can be applied and have been thoroughly investigated in the literature. Active methods such as pulsatile flows (Timité et al., 2011) or electro-magnetic fields (Brancher and Leprevost, 2004) have been studied and lead to good results. Another mixer for applications such as food engineering is studied by Metcalfe and Lester (2009). Rotating specific areas of the tube wall helps increasing the mixing for low Reynolds numbers. However those active methods require an external energy source that, when added to the mechanical energy cost, penalises their global performances. Thus the development of passive devices that rather use their own geometries to change the flow can be a less costly and more effective solution.

In laminar regimes, one of the best ways to significantly enhance the mixing consists in organising the flow structure with the aim of producing chaotic advection. This specific flow regime was first introduced by Aref (1984), Aref (1990), and Aref (2002). The flow is said to be chaotic when it is deterministic in an Eulerian point of view whereas it presents a stochastic behaviour in a Lagrangian point of view. Chaotic advection may appear in two dimensions unsteady flows or in three dimensions flows, because of the device geometry itself or

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Nomenclature

T	temperature (K)
D	diameter (m)
D_h	hydraulic diameter (m)
A_e	dimensionless deformation amplitude
a_i	dimensionless major axis of the elliptic cross-section
b_i	dimensionless minor axis of the elliptic cross-section
p_i	dimensionless pitch of the swirled core
Re	Reynolds number based on D_h
Pr	Prandtl number
U	flow velocity ($\mathrm{m\,s^{-1}}$)
x	coordinate in the streamwise direction (m)
y	coordinate in the radial direction (m)
z	coordinate in the radial direction (m)
M	mixing rate (%)
d	distance between two tracked particles
Greek symbols	
σ_X	standard deviation of X
λ_e	dimensionless deformation wavelength
Subscripts	
0	at the inlet
x	longitudinal
mean	mean
e	external
i	internal
R	radial
ω	tangential

the applied boundary conditions (Lefèvre et al., 2003). Among the characteristics of a chaotic system, at least one out of the three following conditions must be met: (i) a strong sensitivity to boundary conditions, (ii) contraction and stretching zones creating characteristic horseshoe maps, (iii) specific types of Poincaré sections (Castelain et al., 2001). Several tube geometries following this principle have therefore been studied such as, for example, tubes using alternate Dean flows (Mokrani et al., 1997; Jiang et al., 2004; Jones et al., 1989; Chagny et al., 2000; Habchi et al., 2009; Kumar and Nigam, 2007; Sui et al., 2012). To enhance the heat transfer, Raynal and Gence (1997) showed that chaotic mixing can be more efficient than turbulent mixing in some situations. Their work shows in particular that chaos, introduced for example by successive stretchings and foldings (as stated by the baker's theory (Wiggins and Ottino, 2004)), can widely amplify the part played by diffusion. Therefore, the mixing is accelerated with a lower energy cost compared with turbulence. In order to increase the mixing and create chaotic advection, a technique consists in superimposing different vortical macro-structures patterns in the flow (Jiang et al., 2004). That is precisely the strategy chosen in the present study.

Wall deformations are a well-known enhancement technique and can help creating a secondary flow that increases the transfer in laminar flow regime. Numerous types of deformations can be found such as corrugated tubes studied for laminar flows (Barba et al., 2002; Rainieri and Pagliarini, 2002) or constant cross-section area tubes such as the elliptic twisted tube (Tan et al., 2012). Among tubes that keep a nearly constant cross section to limit recirculation zones and the increase of pressure drops, interesting flow patterns can be created in geometries such as the successive alternate deformed tube (Harion et al., 2000) or the staggered oval tube (Chen et al., 2004, 2006). Radial centripetal and centrifugal movements are created by the wall deformations, thereby producing streamwise vortical structures and increasing the transfer within the flow.

Adding inserts or a core inside a tube also modifies the flow dynamics and its transfer properties. Vortical macro-structures are specifically created. A commonly used insert geometry is the twisted tape insert that generates a swirled flow (Yerra et al., 2007). Numerous variations around this kind of internal geometry have been studied for static mixers adaptations such as the already commercialised Kenics mixer (Thakur et al., 2003; Meijer et al., 2012; Saatdjian et al., 2012) or the dough mixer with a deformed core (Hosseinalipour et al., 2013).

In the present article, the superposition of a radial flow pattern due to alternate wall deformations and a swirled flow resulting from an inserted swirled core tube has been studied: this type of annular tube geometry leads to efficient mixing. The analysis has been made by comparing numerically each deformation type separately and with their combination.

The assessment of the mixing has been performed by using several tools. Firstly, as done by Saatdjian et al. (2012) and Mokrani et al. (1997), the temperature has been used as a passive scalar, to mark the mixing state and its standard deviation evolution along the tube length has been studied. The mixing rate has been defined for a given tube length x as:

$$M(x) = \frac{\sigma_0^T/T_{mean,0} - \sigma_x^T/T_{mean,x}}{\sigma_0^T/T_{mean,0}} \tag{1}$$

σ_x^T is the standard deviation of fluid temperature calculated on the tube cross-section taken at the tube length x. The flow structure analysis, with a focus on its potential chaotic behaviour, has been conducted by a Lagrangian analysis using passive tracers trajectories tracking and the representation of Poincaré sections similarly as in previous works (Thakur et al., 2003; Jiang et al., 2004; Jones et al., 1989; Habchi et al., 2009; Sui et al., 2012; Lefèvre et al., 2003).

2. Geometry and model

2.1. Tubes geometries

Three coaxial tube geometries have been defined to investigate separately the flow structure of two kinds of deformations and their combination. The first geometry aims at analysing the flow created in an annular tube with external wall deformations and a simple smooth cylindrical core. Such a Deformed External Tube (*DET*) is presented in Fig. 1(a). The second tube configuration gives informations about the flow induced by a swirled core inserted in a simple smooth cylindrical tube. It is referenced as the SC configuration and is shown in Fig. 1(b). The third tube consists of the superposition of those two specific flow patterns, as can be seen in Fig. 1(c). This deformed external annular tube with a swirled core will be further referenced as *DETSC*.

For the *DET* geometry, while the external tube cross-section shape gradually varies, its area remain constant to avoid recirculation zones. Such a deformation type has already been