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Walking dynamic similarity induced by a combination of Froude and Strouhal dimensionless numbers: Modela-w

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ABSTRACT

The aim of this study was to assess the accuracy of a new dimensionless number associating Froude (Nfr) and Strouhal (Str) called Modela-w to induce walking dynamic similarity among humans of different sizes. Nineteen subjects walked in three experimental conditions: (i) constant speed, (ii) similar speed (Nfr) and (iii) similar speed and similar step frequency (Modela-w). The dynamic similarity was evaluated from scale factors computed with anthropometric, temporal, kinematic and kinetic data and from the decrease of the variability of the parameters expressed in their dimensionless form. Over a total of 36 dynamic parameters, dynamic similarity from scale factors was met for 11 (mean $r = 0.51$), 22 (mean $r = 0.52$) and 30 (mean $r = 0.69$) parameters in the first, the second and the third experimental conditions, respectively. Modela-w also reduced the variability of the dimensionless preceding parameters compared to the other experimental conditions. This study shows that the combination of Nfr and Str called Modela-w ensures dynamic similarity between different-sized subjects and allows scientists to impose similar experimental conditions removing all anthropometric effects.

1. Introduction

Dynamic similarity studies allow for the comparison of the locomotion between species [1] and the reduction of different-sized subjects inter-individual variability [2,3]. The concept of dynamic similarity states that two systems are dynamically similar when all the lengths, the masses and the times of the smaller are equal to those of the taller by multiplying them with the same coefficients C_L, C_M and C_T which represent the basis scale factors. Therefore, all other mechanical scale factors are determined from the combination of the basis scale factor, such as speed, force or impulse (Table 1). The main challenge when studying locomotion in different-sized specimens is to define experimental conditions enabling dynamic similarity to be observed. Dynamic similarity between two systems is met in particular conditions, which depend on the nature of the force involved.

The Inverted Pendulum (IP) consists of a body mass represented at the Center of Mass (CoM) oscillating at the end of a massless rigid segment [4], and is mainly used to model walking because of the out-of-phase relationship between potential ($E_P = mgh$; m the mass, g the gravity and h the CoM height) and kinetic ($E_K = 0.5mv^2$; v the speed) energies. The ratio of E_K and E_P can be simplified to Froude number ($\text{Nfr} = 2E_K/E_P$). Many studies consider Nfr as the normalized speed (dimensionless speed) and use it to compare different species [1] or subjects speeds reported to a characteristic length. Another use of Nfr is to first determine Nfr fractions, then walking speed adapted to body length [2,3]. These authors demonstrate that it is a good mean to establish dynamic similarity between different-sized subjects. Note that this method aims to impose a same Nfr fraction to the subject (i.e. a same energy ratio).

However, more recent studies have shown the limits in considering the walking gait as an IP [5,6]. These works suggested the presence of an elastic phenomenon in walking. Therefore, the Spring Mass Model (SMM) seems to be adapted [7]. SMM, taking into account an elastic component, elastic energy ($E_E = 0.5k\Delta l^2$ with k for the spring stiffness and Δl for the variation of the spring length), could play an important role in mechanical energy conservation in walking as it has been shown to do in running [8].

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Table 1
Units, dimensions and predicted scale factors of kinetic parameters.

Parameters	Units (SI)	Dimensions	Predicted scale factors	Dimensionless parameters
CoM height (l)	m	L	C_L	
Body mass (m)	kg	M	C_M	
Speed (v)	m s^{-1}	LT^{-1}	$C_L C_T^{-1}$	Nfr
CoM oscillation frequency (f)	s^{-1}	T^{-1}	C_T^{-1}	Str
Time (TC and TPPF)	s	T	C_T	Time^D = time × f
Force (DPF and BPF)	N	MLT^{-2}	$C_M C_L C_T^{-2}$	Force^D = force/(mlf^2)
Impulse (VI, BI and PI)	N s	MLT^{-1}	$C_M C_L C_T^{-1}$	Impulse^D = impulse/(mgf)
Rate (LR)	N s^{-1}	MLT^{-3}	$C_M C_L C_T^{-3}$	Rate^D = rate/(mlf^3)
Length (SL)	m	L	C_L	Length^D = length/l
Angle (ankle, knee and hip)	Rad			Angle

C_L and C_M were defined by the subject's anthropometry whereas C_T was determined by the experimental conditions. TC, time of contact; TPPF, time to propulsive peak force; DPF, damping peak force; BPF, braking peak force; VI, vertical impulse; BI, braking impulse; PI, propulsive impulse; LR, loading rate; SL, step length.

The SMM is a conservative system: $E_K + E_P + E_E$ = constant. Besides the energy transfer that occurs at the CoM during running (Modela-r; [9]), in walking a transfer occurs from E_K to E_P and E_E in the first half of stance, and then, conversely in the second half. Indeed, upon reaching the apex of CoM trajectory, (E_P increases) the spring is under compression (E_E increases) and the CoM speed is reduced (E_K decreases); then, the CoM returns to its initial height (E_P decreases), the spring length recovers its rest length (E_E decreases) and the CoM speed rises (E_K increases). We propose to develop the energy ratio as follows and to name it Modela-w:

$$\text{Molela-w} = \frac{E_K}{E_P + E_E} = \left(\frac{mgh + (1/2)k\Delta l^2}{(1/2)mv^2}\right)^{-1}$$
$$= \left(2\frac{mgh}{mv^2} + \frac{k\Delta l^2}{mv^2}\right)^{-1} = \left(2\frac{gh}{v^2} + \frac{f_0^2\Delta l^2}{v^2}\right)^{-1} \quad (1)$$

with $f_0 = \sqrt{k/m}$; (v^2/gh) is usually referred to as Nfr.

As the authors [1,2] suggested, given a concomitant use of Nfr and Strouhal (Str = frequency × length/velocity) to induce dynamic similarity in running which shares the same SMM with walking, Modela-w can be expressed in Nfr and Str terms:

$$\frac{E_K}{E_P + E_E} = \left(2\frac{gh}{v^2} + \frac{f_0^2\Delta l^2}{v^2}\right)^{-1} = (2\text{Nfr}^{-1} + \text{Str}^2)^{-1} \quad (2)$$

Thus, Modela-w reveals a combination of Nfr and Str: Modela-w = 1/(2Nfr^{-1} + Str^2) and is adapted to explain the energy transfer that occurs during walking if an elastic component is considered.

Our study aims to ensure dynamic similarity among different-sized subjects using a combination of Nfr and Str for walking through the introduction of Modela-w as a dimensionless number issued from the energy transfer at the CoM. The main idea is to determine similar conditions for different-sized subjects inducing similar behaviors, and therefore the decrease of inter-subject variability of dimensionless parameters.

2. Methods

2.1. Population

Nineteen healthy men (n = 19) took part in this study after signing an informed consent document. They were chosen so as they had the same density index (mass/height^3) to respect the proportionality law inducing the tallest as the heaviest and vice versa. Their characteristics were (mean ± SD [min; max]): age 23 ± 5 [18; 36] years, height 1.79 ± 0.07 [1.68; 1.94] m, mass 80.7 ± 11 [64; 102.9] kg and density index 14.01 ± 0.42 [13.27; 14.85] kg m^{-3}. All were familiarized with walking test performed on a treadmill. The CoM height (l_i) was determined from the ith subject's anatomic position ($i \in [1, n]$) with the anthropometric model of Zatsiorsky [10]. In order to assess leg joint angles, the center of rotation of the hip was determined using the SCoRE method [11].

2.2. Experimentation

2.2.1. General procedure

For three dimensional kinematic analysis, 42 reflective markers were fixed on subject bone landmarks [12,13]. They performed walking tests with speed and/or step frequency determined from Nfr and Str. To define the experimental conditions, Nfr and Str respectively equaled to v^2/gl and to fl/v; with g the gravity, f the frequency oscillation of the CoM, l the CoM height and v the forward speed. Experimentation was realized on a treadmill (PF 500 CX, PRO FORM, Villepreux, FRANCE) mounted on a large forceplate sampled at 1 kHz (AMTI, Watertown, MA, USA) in a space surrounded by twelve optoelectronic cameras sampled at 200 Hz (VICON, Oxford's metrics, Oxford, UK). After a familiarization period, the subjects had to perform three trials per walking test [14] that were repeated in different experimental settings.

2.2.2. Experimental conditions (EC)

The experimentation consisted of three steps detailed in Fig. 1 and below: (i) same speed (EC_{SPEED}), (ii) similar speed (i.e. same Froude, EC_{NFR}) and (iii) similar speed and similar frequency (i.e. same Nfr and same Str, then same Modela-w, EC_{MOD}).

2.2.2.1. EC_{SPEED}. The subjects performed four walking stages with speeds set at 0.56, 1.11, 1.67 and 2.22 m s^{-1} (Eq. (3)) corresponding to increments of 0.556 m s^{-1} (=2 km h^{-1}). The procedure presented below was repeated for each speed stage. The first experimental condition consisted of setting the same speed for all subjects:

$$v_i = \text{constant} = v \quad (3)$$

2.2.2.2. EC_{NFR}. The second experimentation step entailed imposing similar velocities. A mean $\overline{\text{Nfr}}$ was computed from EC_{SET}. Then, similar velocities for each subject were determined from $\overline{\text{Nfr}}$ (Eq. (4)) I prefer v$_i$ rather than m$_i$ because m represents the mass and v the velocity

$$v_i = \sqrt{\overline{\text{Nfr}} g l_i} \quad (4)$$

2.2.2.3. EC_{MOD}. The third experimentation step consisted of imposing similar velocities (Eq. (4)) and similar frequencies (induced by a metronome). A mean $\overline{\text{Str}}$ was computed from EC_{NFR}. Then, similar frequencies for each subject were determined from $\overline{\text{Str}}$ (Eq. (5))

$$fsim_i = \overline{\text{Str}}\frac{m_i}{l_i} \quad (5)$$