Gait & Posture 41 (2015) 389–394

Contents lists available at ScienceDirect

Gait & Posture

journal homepage: www.elsevier.com/locate/gaitpost

The strengths and weaknesses of inverted pendulum models of human walking

Michael McGrath [a,*], David Howard [b], Richard Baker [a]

[a] School of Health Sciences, University of Salford, M6 6PU Salford, UK
[b] School of Computing, Science and Engineering, University of Salford, M5 4WT Salford, UK

ARTICLE INFO

Article history:
Received 9 December 2013
Received in revised form 10 September 2014
Accepted 22 October 2014

Keywords:
Inverted pendulum
Gait
Walking
Modelling

ABSTRACT

An investigation into the kinematic and kinetic predictions of two "inverted pendulum" (IP) models of gait was undertaken. The first model consisted of a single leg, with anthropometrically correct mass and moment of inertia, and a point mass at the hip representing the rest of the body. A second model incorporating the physiological extension of a head–arms–trunk (HAT) segment, held upright by an actuated hip moment, was developed for comparison. Simulations were performed, using both models, and quantitatively compared with empirical gait data. There was little difference between the two models' predictions of kinematics and ground reaction force (GRF). The models agreed well with empirical data through mid-stance (20–40% of the gait cycle) suggesting that IP models adequately simulate this phase (mean error less than one standard deviation). IP models are not cyclic, however, and cannot adequately simulate double support and step-to-step transition. This is because the forces under both legs augment each other during double support to increase the vertical GRF. The incorporation of an actuated hip joint was the most novel change and added a new dimension to the classic IP model. The hip moment curve produced was similar to those measured during experimental walking trials. As a result, it was interpreted that the primary role of the hip musculature in stance is to keep the HAT upright. Careful consideration of the differences between the models throws light on what the different terms within the GRF equation truly represent.

1. Introduction

As far as the authors have been able to ascertain the first mention of the term "inverted pendulum" (IP) as a model of the stance phase of walking was by Cavagna et al. [1] although similar concepts can be traced much earlier [2–4]. More recently, the IP has formed the basis of a growing body of work associated with the Dynamic Walking movement [5] which is based on principles first elucidated by Mochon and McMahon [6] and subsequently by Tad McGeer [7,8]. Recent work of this group has tended to focus on the transitions from one step to the next [9–11]. The group, as well as other researchers, have also presented many extended versions of IP models including springs, dampers, telescopic actuators, additional segments and joints [12–16].

This work has focussed on energetics and stability whereas the kinematics and kinetics of movement are more relevant to most clinical biomechanists and is less well understood. The mechanics of the IP mechanism itself (as opposed to the transitions), were presented briefly by Anderson and Pandy [appendix of 17], and gave a brief description of the ground reaction force (GRF) under an IP. A more comprehensive comparison with gait data by Buczek et al. [18] concluded that the IP predicts the anterior velocity of the whole body centre-of-mass (CM) and anterior component of the GRF reasonably well but not the vertical components.

The aim of this paper is thus to build on the work of Buczek et al. [18] in extending the ideas of the Dynamic Walking Group into the domain of clinical biomechanics. This includes extending their analysis to include fast and slow walking speeds, and extending the IP model to include a hip joint controlled by a joint actuator in such a way as to maintain an upright head–arms–trunk (HAT) segment. This will be done by starting the HAT segment with zero angular velocity and calculating the moment so that there is zero angular acceleration. Doing this throughout the simulation will ensure there is no angular motion of the HAT. Whilst this is unlikely to affect the overall dynamics of the system (it is still a one degree-of-freedom system) it will

* Corresponding author. Tel.: +44 7531445456.
E-mail address: m.p.mcgrath@edu.salford.ac.uk (M. McGrath).

http://dx.doi.org/10.1016/j.gaitpost.2014.10.023
0966-6362/

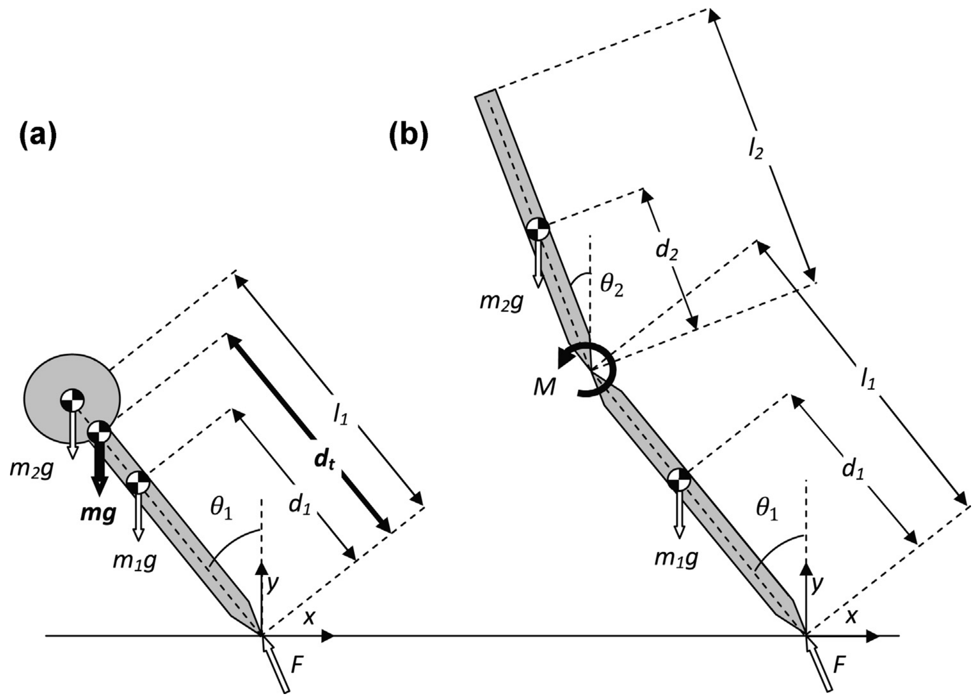

Fig. 1. Free body diagrams for (a) the IP model (including the calculation approximations in bold) and (b) the HAT model of the stance phase of walking.

allow an investigation of the effects of the hip flexor and extensor muscles. There is considerable current interest in the decomposition of the GRF to investigate the function of different muscles [17,19,20] and the analysis of the IP model has been extended to evaluate the contribution of the hip actuator to the GRF.

2. Method

Fig. 1a shows the free body diagram for the simple IP model. The inertial properties of the IP have been altered from the previous models [18] in that the *'leg'* has been assigned a mass (m_1), with CM at a point a given distance (d_1) from the pivot, and moment of inertia (I_1). This change was motivated by the desire for the mass properties of the leg to be the same in both models to avoid an associated confounding effect. The mass of the rest of the body (m_2) acts through a single point at the *'hip joint'*, a given distance (l_1) from the pivot. The mass at the hip has no moment of inertia which is equivalent to assuming that it is concentrated at a single point as assumed in other simple IP models [18,21]. The anterior-posterior direction is defined as the x axis and the vertical direction is defined as the y axis.

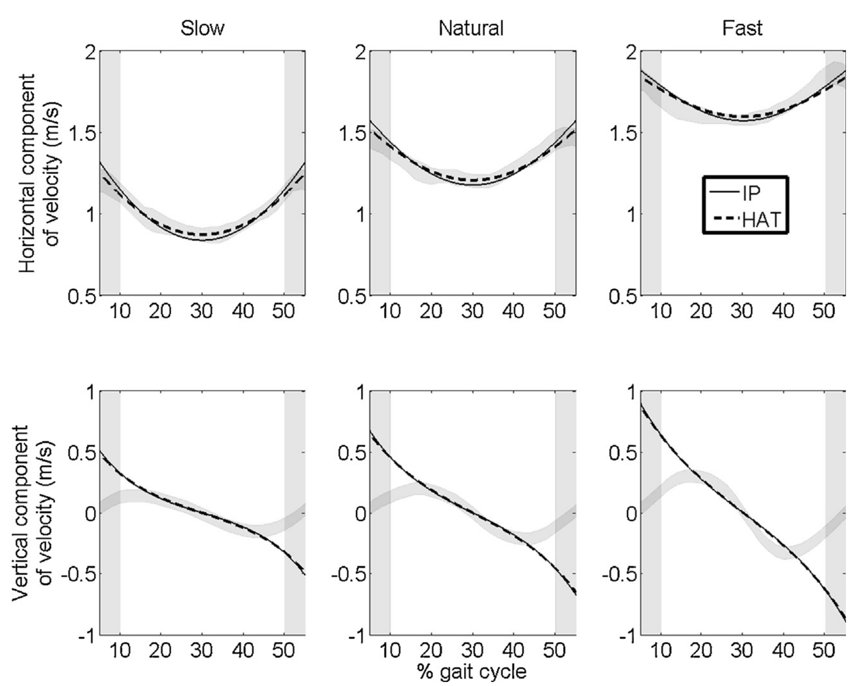

Fig. 2. The linear velocity components for the IP and HAT models at different walking speeds. The shaded areas indicate experimental data and double stance periods. The simulations represented the half-gait cycle from the middle of one double support phase (at 5% of the gait cycle) to the middle of the next (at 55% of the gait cycle).