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A "principal stress cap" model for stresses in a circular silo with an off-centre circular core: Finite core models, including filled silos, incipient flow and switch stresses

A.J. Matchett[a,*,1], P.A. Langston[b], D. McGlinchey[c]

[a] University of Teesside, United Kingdom
[b] Faculty of Engineering, University of Nottingham, United Kingdom
[c] School of Engineering, Glasgow Caledonian University, United Kingdom

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ABSTRACT

Stresses have been modelled in a silo with offset centre of stress and finite circular core, using the methodology developed by Matchett et al. (2015). Several types of core-annulus stress interactions have been proposed and some of the problems in the original Virtual Core model have been ameliorated. However, the selection of the most appropriate model is limited by lack of data on internal stress distributions within silos and the observation that different internal structures can give similar wall stress values.

Passive systems with convex stress cap and active stress systems with concave stress cap have been modelled. In order to keep wall shear stresses and internal stresses below the yield limits, the model suggests that deep, completely filled silos would have very small values of wall arc normal angles, β_c and β_w, and stress eccentricity, *Ecc*. Deep, filled silos with high stress eccentricity and large wall normal angles are not viable.

Incipient flow and the stress switch have been simulated. Output data suggest wide variation in wall stresses both axially and azimuthally are possible, at high stress eccentricities, which would have structural implications.

1. Introduction

Matchett et al. (2015) developed a three-dimensional model of asymmetrical stresses in a cylindrical silo and the present paper is a continuation of the work presented there.

Silos with eccentric discharge have long been known to give problems of flow and structural integrity, due to variations in wall stresses both vertically and azimuthally (Sadowski and Rotter, 2011; bulk-online forum, 2015). Workers take encouragement from Carson's assertion (Carson, 2000) that stress and flow eccentricity is one of the major causes of silo failure.

There is an extensive body of literature in this field and several excellent reviews (for example, Sielamovicz et al., 2010).

Since the previous paper, studies of eccentricity have continued to be published:

Recent publications in this field can be divided into 3 broad, often overlapping categories:

Structural analysis, including vessel stresses, failure and buckling.

Experimental studies: these may be model-based or studies of full-scale silos.

* *Corresponding author at*: 15 Rufford Close, Guisborough, Redcar & Cleveland, England TS14 7PU, United Kingdom. Tel.: +44 1287 637262.
E-mail address: matchett15@ntlworld.com (A.J. Matchett).
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Nomenclature

Symbols

a_1	relates projected circle radius R_1 to X; $R_1 = a_1X + a_1$ ’
a_1'	constant relating R_1 to X – see a_1 (m)
a_2	differential of projected circle centre $a_2 = \frac{\partial O_X}{\partial X}$
D	D/DX and D/DZ are differentials along the principal stress paths for changes in X and Z, respectively
e_1	angle used in the calculation of R_2 (rad)
E	factor relating rotation in the horizontal plane to rotation on the λ-plane
h	inner circle offset (m)
k	ratio of wall vertical to normal stress: Janssen model
K_w	ratio of σ_1/σ_3 at the wall
M	conical yield function parameter
r_1	inner circle radius (m)
r_2	outer circle (silo) radius (m)
R_1	radius of projected horizontal circle of principal stress path (m)
R_2	radius of principal stress cap at a general point (m)
R_{20}	value of R_2 at $\varepsilon_1 = 0$ (m)
$R_{2\pi}$	value of R_2 at $\varepsilon_1 = \pi$ (m)
w_1	arc length along ψ_1-line, seen as $\frac{\partial w_1}{\partial X}$ (m)
w_2	arc length along ψ_2-line, seen as $\frac{\partial w_2}{\partial Z}$ (m)
x	x-axis co-ordinate (m)
X	intercept of projected horizontal surface with x-axis (m)
X_o	minimum value of X (m)
X_{max}	Maximum value of X (m)
x_1, x_2, x_3	Local Cartesian co-ordinates coincident with directions of principal stress
y	y-axis co-ordinate (m)
z	z-axis co-ordinate (m)
Z	value of z for the inner radius of the principal stress cap (m)
Z_o	value of Z at the point of boundary conditions (m)
β_c	angle of circular arc to normal at inner core (rad)
β_w	angle of circular arc to normal at wall (rad)
ε_1	angle from x-axis in the horizontal plane (rad)
ε_2	angle from the vertical in the x–z plane at $\varepsilon_1 = 0$, rotated along the elliptical, principal stress path (rad)
ε_3	angle from the vertical slope of the principal stress cap surface as seen from ε_1 (rad)
γ_1	slope of principal stress, σ_3 at the wall (Pa/m)
γ_2	slope of principal stress, σ_1 at the wall
ϕ	angle of internal friction. A nominal value of 30° has been used (rad)
λ	characteristic slope of principal stress path ellipse when projected onto the x–z plane (rad)
μ_w	coefficient of wall friction. A nominal value of 0.3 has been used.
η	surcharge friction factor
θ_{lim}	limiting value of wall arc angle (rad)
ζ_{lim}	limiting value of plane of yield (rad)
ψ_1	angle of ψ_1-line to x-axis on the horizontal plane (rad)
ψ_2	angle of ψ_2-line to vertical–principal stress path for changes in Z (rad)
ρ	bulk density of the bulk solid in the silo (kg/m^3)
σ_1	principal stress in x_1 direction (Pa)
σ_2	principal stress in x_2 direction (Pa)
σ_3	principal stress in x_3 direction (Pa)

Modelling: the use of DEM, FEM and continuum models to predict bulk behaviour.

Investigations may include stresses generated, flow patterns and/or both. It is generally accepted that flow patterns affect stresses during discharge (Sielamovicz et al., 2010).

Structural analysis: There continues to be a lively interest in vessel structures. Buckling has been analysed in silos of different methods of construction (Wojcik and Tejchman, 2015; Sondej et al., 2015). Sondej et al. (2015) considered the implications of their work on the design codes.

Experimental studies: Sielamovicz et al. (2015) continued their studies of eccentric flow patterns in a “2-d” model silo, following on from Sielamovicz et al. (2010, 2011). On a much larger scale, Ramirez-Gomez et al. (2015) measured stresses in the roof sections of an agricultural silo.

Modelling: Wang et al. (2015) used their FEM system to analyse a flat-bottomed silo, predicting stresses, including a comparison with experimental data. Wojcik and Tejchman (2015) used a hypoplastic constitutive model within a FEM algorithm for sand to generate bulk solid stresses in their work on buckling, illustrating the fluidity of categories above. Wang et al. (2015) used a macroscopic elasto-plastic constitutive model with linear Drucker–Prager criterion and a perfect plastic flow rule.

The authors’ own paper (Matchett et al., 2015) is discussed below.

It is useful to differentiate between eccentric systems in which the core or flow channel touches the silo wall, and those systems in which the core/flow channel is eccentric but does not touch the wall. Several analyses model systems with the core touching the wall (Sadowski and Rotter, 2011; Sielamovicz, 2010, 2011, 2015). Eurocode 1 (2006) is based upon this approach. The model of Matchett et al. (2015) uses a core that does not touch the wall. The geometrical complexities make this an issue for further development.

Matchett et al. (2015) developed a three-dimensional model of asymmetrical stresses in a cylindrical silo with an inner, offset, circular core (see Fig. 1). The model was based upon the principal stress cap concept of Enstad (1975). The reader is referred to the original paper for details of the model (Matchett et al., 2015)

The output from the model was compared to wall stress data from a DEM simulation for a completely filled silo ($r_1 = 0$), with reasonable agreement. There were problems at $X = 0$, ($R_1 = 0$) leading to a discontinuity in σ_3 and excessive stress peak values around $\varepsilon_1 = 0$ in deep beds with high eccentricity (*Ecc*). The hypothesis of a virtual core was proposed, but this was not entirely satisfactory. It was suggested that this problem might be overcome by use of an actual, finite core, rather than the virtual core.