chemical engineering research and design xxx (2015) xxx–xxx

Contents lists available at ScienceDirect

Chemical Engineering Research and Design

journal homepage: www.elsevier.com/locate/cherd

Eigenvalue spectrum versus energy density spectrum in a mixing tank

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ARTICLE INFO

Article history:
Received 30 July 2015
Received in revised form 9 October 2015
Accepted 14 October 2015
Available online xxx

Keywords:
Proper orthogonal decomposition
Particle image velocimetry
Eigenvalue spectrum
Energy density spectrum
Information entropy

ABSTRACT

The goal of this paper is to relate eigenvalue spectrum issued from POD to energy density spectrum. In the first part of this paper, eigenvalue spectrum issued from POD is plotted. Reconstruction of kinetic energy (KE) and dissipation rate of KE are discussed. 1D energy density spectrum is plotted for complete fluctuating velocity field. Different projections of fluctuating velocities on selected groups of eigenmodes are discussed. In the second part of this paper, 1D longitudinal EDS is reconstructed by the way of information entropy (Ogawa, 2007), as a new perspective. The maximum entropy method (MEM) is used to derive the energy density spectrum (versus wave number) in the inertial range of a turbulent flow.

1. Introduction

P.I.V. (Particle Image Velocimetry) technique is applied in the field of hydrodynamics in a mixing tank (Bugay et al., 2002; Escudié and Liné, 2003; Huchet et al., 2009; Gabelle et al., 2013). Recently, P.O.D. (Proper Orthogonal Decomposition, Berkooz et al., 1993) technique was used to extract more and more information from such huge source of experimental data (Moreau and Liné, 2006; Doulgerakis's PhD, 2010; Doulgerakis et al., 2011; Liné et al., 2013). POD is an efficient technique to process instantaneous velocity fields, enabling to reconstruct the velocity in terms of summation of modes, each mode contributing to the total kinetic energy. P.O.D. is thus a modal decomposition of instantaneous velocity fields. Modes can be derived from the Fredholm eigenvalue integral equation, adapted by Sirovich (1987). Knight and Sirovich (1990) have shown that the POD eigenvalue spectrum (plot of eigenvalues versus mode number I) exhibit $I^{-11/9}$ trend characteristic of the inertial subrange of turbulence. The goal of this paper is to relate eigenvalue spectrum issued from POD to energy density spectrum. In the first part, energy density spectrum will be reconstructed from selected modes issued from POD. In the second part of the paper, the energy density spectrum (or probability density functions PDF of velocity fluctuations) will be derived in turbulent flow from "entropy information" (Arimitsu and Arimitsu, 2002; Verkley and Lynch, 2009).

In this introduction, the "information theory" is shortly presented and Maximum Entropy Method (MEM) is reminded. Secondly, the approach developed by Ogawa (2007) is reviewed. Thus, an expression of the total energy density per unit wave number κ is derived. This approach will be revisited by coupling information theory to information issued from POD analysis.

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http://dx.doi.org/10.1016/j.cherd.2015.10.023

1.1. Eigen value spectrum versus mode number

Referring to Proper Orthogonal Decomposition, Knight and Sirovich (1990) analyzed the eigenvalue spectrum of the two-point velocity correlation tensor. The eigenvalue spectrum is plotted versus mode number I. Recall that the eigenvalue associated to Ith mode represents the kinetic energy allocated to this mode. Knight and Sirovich have shown that, in the inertial range of turbulence, the eigenvalues were a generalization of the energy density spectrum. Thus, they demonstrated that the Ith eigenvalue $\lambda^{(I)}$ can be related to the Ith mode number by a power law $\lambda^{(I)} \propto I^{-11/9}$. Such a behavior has been observed by many experimental and numerical works (De Angelis et al., 2003; Ducci et al., 2007; Doulgerakis, 2010; Housiadas et al., 2005; Handler et al., 2006; Piponniau et al., 2012; Kefayati and Poepping, 2013; Calaf et al., 2013 among others). In addition, Knight and Sirovich related the Ith wave number κ_I to the Ith eigen mode by the relation $I \propto {\kappa_I}^3$. This trend was confirmed by Liné et al. (2013) from derivation of length scales associated to each eigenvector and more recently by Tang et al. (2014). In the first part of this paper, the eigenvalue spectrum will be presented and discussed. Another way to reconstruct energy density spectrum consisting in applying information entropy to velocity data will be develop in the second part.

1.2. Information theory

Information theory has been developed to quantify the amount of information that is contained in the observation of an event having a probability p (Beck, 2009). The average amount of information, classically noted $H(p)$, is obtained as follows:

$$H(p) = \sum_{i=1}^{N} p_i \log\left(\frac{1}{p_i}\right) \tag{1}$$

where the variable $H(p)$ is called "information entropy". In other words, the "information entropy" of a probability distribution is the value of the amount of information of the whole distribution.

1.3. Maximum entropy method

The maximum entropy method is based on the assumption that the probability distribution that maximizes the "information entropy" is the most expected to occur (Pope, 1979; Martyushev and Seleznev, 2006). This procedure may be fulfilled as follow: let's look for a probability distribution $p(x)$ that maximizes the "information entropy" $H(p)$ given a finite number of moments (Bandyopadhyay et al., 2005). By introducing Lagrangian multipliers λ_m, one can define the Lagrangian functional F:

$$F = H + \sum \lambda_m \left[\int x^m p(x)dx - \mu_m\right] \tag{2}$$

The maximum entropy method can be understood so as to maximize the Lagrangian function given by Eq. (2).

1.4. Application to turbulent flow

The book of Ogawa (2007) has been dedicated to the application of maximum entropy method to chemical engineering in general and turbulent phenomena in particular. In this book, it has been shown (page 14) that if a variable x takes positive values and if its average value (first moment) is fixed as A, thus one can write the two first moments of the distribution as:

$$\int_0^\infty p(x)dx = 1 \quad \int_0^\infty xp(x)dx = A \tag{3}$$

In this case, the functional maximum (Eq. (2)) can be expressed by:

$$\frac{\partial}{\partial p}\int [-p(x)\log(p(x))]\,dx + \lambda_1 \frac{\partial}{\partial p}\left[\int p(x)dx - 1\right] + \lambda_2 \frac{\partial}{\partial p}\left[\int xp(x)dx - A\right] = 0 \tag{4}$$

After calculation (Ogawa book, page 16), it corresponds to a maximum value of the "information entropy" which is equal to:

$$H_{max} = \log(eA) \tag{5}$$

and the probability distribution that maximizes the "information entropy" is derived as:

$$p(x) = \frac{1}{A}\exp\left(-\frac{x}{A}\right) \tag{6}$$

Ogawa (2007) applied the maximum entropy method to derive energy density spectrum of turbulent flows. Following Ogawa, "the ESD is discussed base on the uncertainty regarding the wave number of fluctuation that is selected" (Ogawa book, p 102). Consider that the ESD of the Ith-eddy group is probability density function p. This probability is thus defined as:

$$p_I = \frac{E_I(\kappa)}{u'^2_I} \tag{7}$$

where κ is the wave number, u'^2_I is the turbulent kinetic energy of the Ith-eddy group and $E_I(\kappa)$ is the energy density per unit scalar wave number and associated to Ith-eddy group. Following MEM (Eq. (6)), the probability distribution that maximizes the "information entropy" as well as the total energy density per unit wave number κ are given:

$$\frac{E_I(\kappa)}{u'^2_I} = \frac{1}{\kappa_I}\exp\left(-\frac{\kappa}{\kappa_I}\right) \quad \frac{E(\kappa)}{\overline{u'^2}} = \sum_{i=1}^{n}\frac{P_I}{\kappa_I}\exp\left(-\frac{\kappa}{\kappa_I}\right) \tag{8}$$

where $P_I = \left(u'^2_I/\overline{u'^2}\right)$. In his book (Ogawa book, page 102), Ogawa proposed to relate the eddy scales κ_i and the associated velocity variance P_i as

$$\frac{\kappa_{I+1}}{\kappa_I} = \frac{1}{\alpha} \quad \text{and} \quad \frac{P_{I+1}}{P_I} = \frac{u'^2_{I+1}}{u'^2_I} = \frac{1}{\beta} \tag{9}$$

The final expression of the total energy density is (Ogawa book, page 103):

$$\frac{E(\kappa)}{\overline{u'^2}} = \frac{P_1}{\kappa_1 \sum_{i=1}^{n}(1/\beta)^{i-1}}\sum_{I=1}^{n}\left[\left(\frac{\alpha}{\beta}\right)^{i-1}\exp\left(-\alpha^{I-1}\frac{\kappa}{\kappa_1}\right)\right] \tag{10}$$

Please cite this article in press as: Liné, A., Eigenvalue spectrum versus energy density spectrum in a mixing tank. Chem. Eng. Res. Des. (2015), http://dx.doi.org/10.1016/j.cherd.2015.10.023