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OPTIMAL DETERMINISTIC TRANSFER FUNCTION MODELLING IN THE PRESENCE OF SERIALLY CORRELATED NOISE

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This article addresses the development of predictive transfer function models for non-linear dynamic processes under serially correlated model error. This work is presented in the context of the block-oriented exact solution technique (BEST) for multiple input, multiple output (MIMO) processes proposed by Bhandari and Rollins (2003) for continuous-time modelling and Rollins and Bhandari (2004) for constrained discrete-time modelling. This work proposes a model building methodology that is able to separately determine the steady state, dynamic and noise model structures. It includes a pre-whitening procedure that is affective for the general class of discrete ARMA(p, q) noise (Box and Jenkins, 1976). The proposed method is demonstrated using a simulated physical system and a real physical system.

Keywords: Wiener system; Hammerstein system; predictive modelling; dynamic modelling; block-oriented modelling; ARMA; serially correlated noise.

INTRODUCTION

The noise or error term in a dynamic predictive model if often serially correlated, i.e., related over time. Therefore, in these situations, the predictive ability of a model may be improved from the development and use of an accurate error term model (ETM). Consequently, the purpose of this article is to propose a model development method under autoregressive, moving average (ARMA) noise in the context of the block-oriented method developed by Bhandari and Rollins (2003) for continuous-time modelling and by Rollins and Bhandari (2004) for constrained discrete-time modelling.

In block-oriented modelling, static and dynamic behavior are represented in separate blocks and arranged in a network connected by variables that are either observed or unobserved. The two most basic systems are the Hammerstein system and the Wiener system which are special cases of the more general 'sandwich model' as discussed in Pearson and Ogunnaike (1997). The first block in the Hammerstein system is the static gain function which is typically nonlinear in the inputs. This function then enters the second block consisting of a linear dynamic transfer function that ultimately produces the output response. The Wiener system is similar to the Hammerstein system but reverses the order of the blocks; the Wiener system is shown in Figure 1 for a multiple input, multiple output (MIMO) system decomposed to q multiple input, single output (MISO) blocks (see Nells, 2001). The advantages of the Wiener system over the Hammerstein system are the following: (1) each input has a separate dynamic block; and (2) it addresses nonlinear dynamic behaviour functionally and directly through the blocks connecting the outputs. Note that, block-oriented sandwich models are systems with static nonlinear and linear dynamic blocks arranged in series or parallel connections. Although, in this article, we primarily focus on the Wiener and Hammerstein systems, the methodology that we propose is applicable to block-oriented modelling in general.

Three common sources of serially correlated noise include model mismatch, measurement errors and unmeasured inputs. These sources combine to give the ETM its serially correlated nature. Most of the block-oriented modelling articles found in literature only addresses independently distributed noise or the so-called 'white' noise (e.g., see Gómez and Baeyens, 2004; Hagenblad and Ljung, 2000; Hagenblad, 1999; Bai, 1998; Kalafatis *et al.*, 1997; Westwick and Verhaegan, 1996; Greblicki, 1994; Wigren, 1993). This is an insufficient representation of a 'real' system which will inevitably have serially correlated noise due to these error sources. Hence, this article seeks to overcome this insufficiency with the inclusion of serially correlated noise in block-oriented modelling. Figure 2 is a modification of Figure 1 and illustrates the contributions of unmeasured inputs and measurement error to the error term.

In our literature search we found only a few studies involving serially correlated noise [these included the

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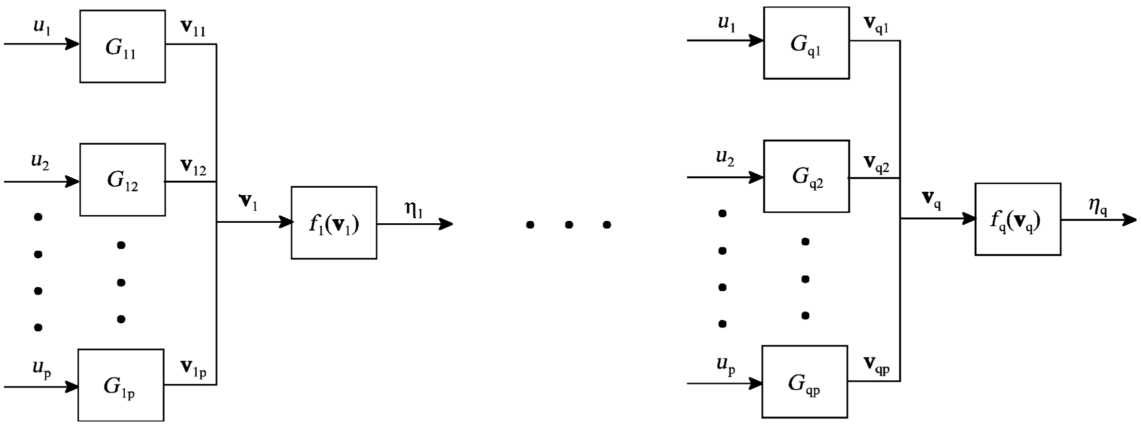

Figure 1. A description of the general MIMO Wiener model structure (decomposed to q MISO blocks) with $i = 1, \ldots, q$ outputs and $j = 1, \ldots, p$ inputs. There is one set of blocks for each of the q outputs. For each set of blocks, each of the p inputs (u_j) passes through a separate linear dynamic block (G_{ij}) and produces an intermediate variable, v_{ij}, that is an element of the vector $\mathbf{v}_i$. Each $\mathbf{v}_i$ passes through a nonlinear static function $f_i(\mathbf{v}_i)$ and generates the output η_i.

works of Cao and Gertler (2004); Zhu (2002); David and Bastin (2001); Chen and Fassois (1992); and Haist *et al.* (1973)] in block-oriented modelling. These studies all employed methods of *simultaneous identification* of the DTFM and ETM structures, rather than separate identification of these structures. Also, only a small fraction of these studies specifically addressed Wiener systems (Zhu, 2002; Chen and Fassois, 1992) or Hammerstein systems (Haist *et al.*, 1973), and none of them involved the modelling of physical systems.

Blocking-oriented modelling of physical systems in the presence of serially correlated noise consists of the determination of three types of model structures: (1) the static or steady-state model (SSM) (f_η in Figure 2); (2) the dynamic deterministic transfer function model (DTFM) (i.e., η in Figure 2); and (3) the dynamic ETM (ε in Figure 2). If the goal is to determine the DTFM that explains the greatest amount of variation in the output, then identification of this model can be quite challenging as it is competing with the ETM for dynamic predictive power. In view of this, we make the following comments. First, the information for determining the DTFM comes from the relationships of the past inputs on the current output. Secondly, the information for determining the ETM comes from the relationships of the past outputs on the current output. Furthermore, past outputs contain a composite of input information that makes the past values of an output variable more information-rich than the past values of any one input variable. Thus, in many situations, it is possible to obtain high predictive accuracy without the use of any (or only a few) input variables. Note that this is the core justification for autoregressive-integrated moving average (ARIMA) modelling (see Box and Jenkins, 1976) which uses no inputs and is a dynamic modelling approach based strictly on past outputs. Therefore, given a transfer function modelling problem where ARIMA modelling alone can be quite effective, one could obtain excellent performance irrespective of the DTFM and its contribution. Consequently, in a dynamic setting, under serially correlated noise, the modeller must be careful not to allow the ETM to take predictive power away from the DTFM when the goal is to obtain the optimal DTFM (i.e., the one with maximum predictive

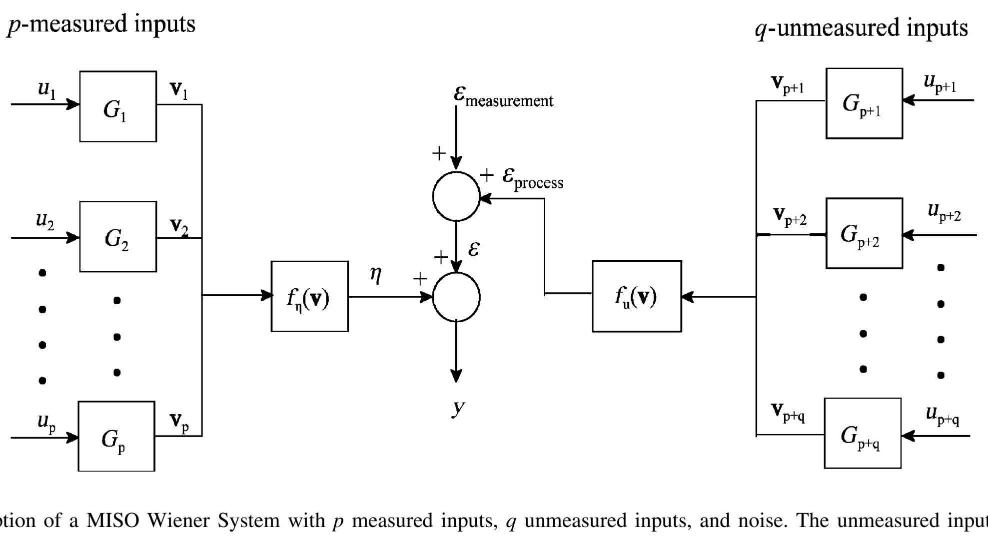

Figure 2. A description of a MISO Wiener System with p measured inputs, q unmeasured inputs, and noise. The unmeasured inputs contribute to the unmeasured process noise, $\varepsilon_{\text{process}}$. The $\varepsilon_{\text{measurement}}$ term represents all the measurement errors. The error term, ε, is equal to $\varepsilon_{\text{process}}$ plus $\varepsilon_{\text{measurement}}$. The output, y, is equal to the exogenous (deterministic) term, η, plus the error (stochastic) term, ε (i.e., the ETM).