PREDICTION OF HEAT TRANSFER COEFFICIENT BASED ON EDDY DIFFUSIVITY CONCEPT

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Abstract: The concept of eddy diffusivity is reviewed, and theoretical comparison study is performed to predict heat transfer coefficient using various proposed models for eddy diffusivity at wide range of Prandtl (7–800) and Reynolds numbers (5000–100 000) in pipe flow. The modified Reynolds analogy equation is employed to start the analysis for predicting the heat transfer coefficient. The influence of *Re*, *Pr* and the distance from the wall (y^+) on the eddy diffusivity and consequently on the heat transfer coefficient and temperature profile is studied and discussed. The results are compared with experimental data. A new expression of thermal eddy diffusivity is been developed by comparing modified form of Reynolds analogy correlation with experimental heat transfer data. Using this new expression more accurate results can be obtained. For future research work, It is interesting to include links to other models and to examine the behaviour of this empirical model in complex flow configurations.

Keywords: eddy diffusivity; heat transfer coefficient; pipe flow; turbulence.

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DOI: 10.1205/cherd06002

0263–8762/07/
$30.00 + 0.00

Chemical Engineering Research and Design

Trans IChemE,
Part A, April 2007

INTRODUCTION

Practically all engineering processes involving fluids depend on the interaction of a fluid with a phase boundary. Fluid friction over extended surfaces, heat and mass transfer to fluids in evaporation, distillation and gas absorption are some of the processes involving the transport of momentum, heat and mass from a phase boundary to a fluid in turbulent flow. The understanding of turbulent transport mechanism and the subsequent ability to predict the relevant transport rates are essential to the development of rational design procedures for various processes. Despite many years of intensive research into turbulent diffusion, it is still poorly understood and can only be rather crudely predicted in many cases (Robert and Webster, 2001). Because of highly complex turbulent flow mechanism, the prediction of the transport rates necessarily involves the formulation of conceptual models which embody many simplifying assumptions (Gutfinger, 1975). Various models proposed in the literature constitute the framework of present day predictive theory and may be broadly divided into three general classes: (1) models based on film theory, (2) models based on eddy or turbulent diffusivity, and (3) models based on the surface renewal concept (Danckwerts, 1951). The development of the theory of transfer of heat and mass between a solid surface and a turbulent fluid has been handicapped by the lack of data on the manner in which the eddy diffusivities vary with distance from the surface. Data on eddy diffusivity very close to the surface are particularly important in order to understand the mechanism of transfer at high Prandtl and Schmidt numbers. The eddy diffusivity for heat may be obtained from temperature profiles and measured heat fluxes. Data very near the wall are difficult to obtain by either of these procedures, since the Pitot tubes and thermocouples employed affect the nature of the flow being studied (Sherwood *et al.*, 1968). Interferometric techniques have been used giving values of eddy diffusivity at y^+ as low as 0.5–1. The various empirical constants introduced in the formulation of the turbulence models must be evaluated through comparison with experimental data (Gutfinger, 1975; Gurniki *et al.*, 2000).

The eddy diffusivity behavior in the viscous sub-layer, damped turbulence layer, and turbulent core affect greatly the rate of heat (or mass) transport between the wall and bulk. Previous studies (Von Karman, 1939; Lin *et al.*, 1953; Deissler, 1955; Van Driest, 1956; Wasan and Wilke, 1964; Rosen and Tragardh, 1995; Meignen and Berthoud, 1998; Wang and Nesic, 2003; Gurniki *et al.*, 2000) showed that the eddy diffusivity is function of many variables such as the distance from the wall (y^+), *Re* and *Pr*.

Over the years there were many models proposed for eddy diffusivity. Most of these models need to be examined against the experimental or theoretical results. For the prediction of heat (or mass) transfer rates many authors (Mizushina and Ogino, 1970; Gutfinger, 1975) assumed the validity of the equality of momentum eddy diffusivity and heat (or mass) eddy diffusivity (i.e., $Pr_t = 1$ or $\varepsilon_m = \varepsilon_H$) and showed that this assumption is a good approximation for most practical applications and exhibits good agreement with experimental data for $Pr \geq 1$.

In present analysis, the assumption of unity turbulent Prandtl number is adopted to estimate the heat transfer coefficient from proposed models for momentum eddy diffusivity. The main objective of this work is to predict the heat transfer coefficient from eddy diffusivity models using a modified form of Reynolds analogy and compare the numerical results with experimental data of some investigators to obtain a more accurate expression for eddy diffusivity that holds for wide range of *Pr* and *Re*.

AN OVERVIEW

Mizushina *et al.* (1964) evaluated the heat transfer coefficients for gas entrainment into the turbulent flow of mercury in horizontal and vertical tubes. After five years, Mizushina and his research team presented their eddy diffusivity model (Mizushina and Ogino, 1970; Mizushina *et al.*, 1971, 1975). The authors presented their expression for eddy diffusivity based on the analysis of experimental results for mass transfer at Schmidt numbers between 800 and 15 000, and that the Sherwood number varies with $\frac{1}{3}$ power of the Schmidt number and about 0.9 power of the Reynolds number at *Sc* between 800–15 000 and *Re* between 3000–80 000. An extended work was made by Ueda *et al.* (1977), where they determined the eddy diffusivity near the free surface by heat-transfer experiments ($Pr \approx 3$) in broad open channel flow with high heat flux at the surface, and observed the eddy diffusivity value decreases to zero in proportion to y^+. Also, they developed another model assuming that the damping of turbulence near the surface can be represented by that of the surface wave motion with a wave length equal to the integral scale of turbulence.

At the same period, there are another research groups such Notter and Sleicher working on eddy diffusivity modelling. Sleicher *et al.* (1970), derived analytical expression to solve the turbulent Graetz problem using an extension of the first order analysis which yields higher approximations for the lower eigenfunctions. Then Notter and Sleicher (1971a) compared their analytical results with computer calculations of the fourth Eigen values and constants for Prandtl numbers in the liquid metal range ($Pr < 0.06$), and Notter and Sleicher (1971b) presented their expression for the eddy diffusivity in the turbulent boundary layer near a smooth wall for fluids of high Prandtl number:

$$\frac{\varepsilon_m}{\nu} = \frac{0.0009y^{+3}}{(1 + 0.0067y^{+2})^{1/2}} \quad 0 < y^+ < 45 \tag{1}$$

This expression leads to acceptable Nusselt numbers when the Prandtl number is of order 1 or greater. But if $Pr > 100$, fully developed Nusselt numbers calculated with above eddy diffusivity expression are correlated within 4% by the following simple equation:

$$Nu = 0.0149\, Re^{0.88}\, Pr^{1/3} \tag{2}$$

Based on their previous work, Notter and Sleicher (1972), presented new numerical calculations incorporating the information on eddy diffusivities for heat transfer rates for both the entry and fully developed regions of the pipe. The numerical expressions are: (1) uniform wall temperature:

$$Nu = 4.8 + 0.0156\, Pe^{0.85}\, Pr^{0.08} \quad 0.004 < Pr < 0.1 \tag{3}$$

and (2) for uniform wall heat flux:

$$Nu = 6.3 + 0.0167\, Pe^{0.85}\, Pr^{0.08} \quad 0.004 < Pr < 0.1 \tag{4}$$

Local heat transfer coefficients and fully developed temperature profiles were measured for sodium and potassium eutectic mixture in a pipe at uniform wall temperature. Using eddy diffusivity profiles, Nusselt numbers were calculated in pipes at uniform heat flux for liquid metals (Sleicher *et al.*, 1973). Whereas, Sleicher and Rouse (1975) modified the correlating equation for calculating heat-transfer coefficients to constant property fluids in pipes to correlate variable property data.

Also, other researchers that can be acknowledged for their contributions in early period such as: Morkovin (1965), concluded that eddy diffusivities when viewed as properties of quasi-similar fields can account for the observed characteristics of the layer developing within another layer to the accuracy of observations.

Sherwood *et al.* (1968) measured the instantaneous axial and circumferential components of both velocity and turbulent intensity in the wall region (to $y^+ = 0.2$) for water flowing in a 2 in. glass pipe of Reynolds numbers of 8000–50 000. The authors concluded that values of the eddy viscosity which are several times greater than those commonly assumed to apply at $y^+ < 5$ in the development of the analogies between momentum and mass or heat transfer.

Sheriff and O'kane (1971), used a circular duct to measure the eddy diffusivity of mass for turbulent air flow. For the Reynolds number range tested, 1.3×10^4–1.3×10^5, the authors found that the eddy diffusivity of mass in the central region of the tube:

$$\frac{\varepsilon_m}{\nu} = 25.27\, Re * 10^{-4} - 3.82 \tag{5}$$

Maubach and Rehme (1972) provided research survey on the effect of asymmetric turbulent velocity profiles, the locations of maximum velocity and zero shear stress, and the importance of this effect for the calculation of momentum, heat, and mass transfer in non-circular channels as well as for the discussion on universal velocity profiles.

Quaramby and Quirk (1972) conducted experimental study on the variation of the radial and tangential eddy diffusivities of heat and mass in a fully developed turbulent flow in a plain tube.

Townsend (1979) studied the various flow patterns in a plane wake and boundary layer. The author concluded that the eddies contributing most in intensity or Reynolds stress are less variable in form than all the eddies together, and