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A viscous–inertial model of foam drainage

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ABSTRACT

The common factor that links various current methods of estimating drainage rate through a gas–liquid foam is that all losses of pressure due to flow are assumed to be entirely viscous. However, by drawing analogy with liquid flow through a packed bed, it is apparent that, for foams that are relatively wet or have relatively high Galileo number, there is a significant inertial loss. This is further demonstrated by determining, using computational fluid dynamics, the pressure losses at a constant expansion with fluid flow boundaries. A foam drainage equation that accounts for inertial pressure losses is proposed by adapting the functional form of the Ergun equation for pressure loss due to flow in a packed bed. This is tested against forced drainage data for foam stabilised by SDS with a mono-dispersed bubble size distribution from the literature. It is shown that the model accurately predicts the results with the use of only one adjustable constant, which is, in fact, the number of inertial velocity heads lost due to flow through a slice of foam of one bubble radius in thickness.

Keywords: Foam; Drainage; Inertia; Velocity head

1. Introduction

For the past 50 years, significant effort has been expended to attempt to predict and measure the drainage rate from gas–liquid foams. The so-called 'channel-dominated' foam drainage equation was proposed by Leonard and Lemlich (1965); the derivation was repeated by Gol'dfarb et al. (1988) and Verbist et al. (1996). This assumed that all the losses in the foam occurred in the Plateau borders (i.e. channels) and, although Leonard and Lemlich recognised that there could be slip at the channel walls, the latter treatments assumed that the walls were infinitely rigid. A decade ago, Koehler et al. (1999) proposed an alternative theory that assumed the losses occur due to expansion, contraction and bending of the flow at the nodes within the foam and that there are no losses in the Plateau borders because the walls are incapable of supporting a shear stress. Crucially for the development of the arguments in this paper, the losses in the foam were considered to be entirely viscous in nature.

The forced drainage experiment developed by Weaire et al. (1993) has been extensively used to test whether foams apparently drain according to the 'channel-dominated' theory or 'node-dominated' theory, or whether reality is intermediate between these two models. In general, the velocity of the wet front in the forced drainage experiment is plotted, in double-logarithmic form against liquid flowrate delivered to the top of the surface; a slope of around one-third apparently indicates node-dominated drainage, whereas a slope of one-half indicates channel-dominated drainage.

Recognising that losses could, in fact, occur in both the channels and the nodes, Koehler et al. (2000) proposed a foam drainage equation to reflect this, although all losses were assumed to be viscous since they calculated Reynolds numbers, based upon the radius of curvature of the Plateau borders, to be less than one in most cases considered in their study. This expression required the adoption of two adjustable constants to reflect the viscous losses in both the channels and the nodes. Neethling et al. (2002) proposed a foam drainage expression that reflected the geometry of relatively wet foams that utilised two adjustable constants and assumed that all the losses were viscous.

Stevenson (2006), by invoking dimensional analysis, showed that, for a specific surfactant at a specific concentration and if one assumes that losses are entirely viscous, the drainage rate in an isotropic foam expressed as a dimensionless permeability is a unique function of the volumetric liquid

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Nomenclature

a	cross-sectional flow area upstream of the expansion (m^2)
A	cross-sectional flow area downstream from the expansion (m^2)
A_{pb}	Plateau border cross-sectional area (m^2)
Bo	Boussinesq number defined in Eq. (31)
C	a geometrical constant (≈0.402)
C_e	expansion loss coefficient defined in Eq. (2)
C_c	contraction loss coefficient
d	characteristic diameter of packing in a bed (m)
D_h	hydraulic diameter of the Plateau border (m)
g	acceleration due to gravity ($m\,s^{-2}$)
Ga	Galileo number defined in Eq. (29)
j_d	superficial liquid drainage velocity ($m\,s^{-1}$)
K_1	viscous loss coefficient
K_2	inertial loss coefficient
m	dimensionless number used in Eq. (8)
n	dimensionless index used in Eq. (8)
p	dimensionless number used in Eq. (4)
P	static pressure (Pa)
q	dimensionless index used in Eq. (4)
Q	volumetric liquid flow rate ($m^3\,s^{-1}$)
r	radius of curvature of a Plateau border wall (m)
r_b	characteristic mean bubble radius (m)
r_h	hydraulic radius of a Plateau border (m)
Re	channel Reynolds number defined in Eq. (2)
Re_d	foam drainage Reynolds number defined in Eq. (26)
Re_{pb}	Plateau border Reynolds number defined in Eq. (10)
Re_{pore}	pore Reynolds number defined in Eq. (12)
u	absolute liquid velocity ($m\,s^{-1}$)
v_{pb}	absolute average velocity within the Plateau border ($m\,s^{-1}$)
V	vertical component of the average absolute velocity ($m\,s^{-1}$)
V_f	velocity of the wet-front in forced drainage ($m\,s^{-1}$)
x	vertical dimension in direction of the flow (m)

Greek symbols

ε	volumetric liquid fraction in the foam
κ^*	dimensionless permeability
μ	interstitial liquid dynamic viscosity (Pa s)
μ_S	surface shear viscosity (Pa m s)
Π	dimensionless number defined in Eq. (22)
ρ	interstitial liquid density ($kg\,m^{-3}$)
σ	equilibrium surface tension ($N\,m^{-1}$)
Φ	sphericity of packing in a bed

fraction, and that a power–law relationship fits forced drainage data. Thus, such a power–law fit requires two adjustable constants as did the models of Koehler et al. (2000) and Neethling et al. (2002). In fact, Koehler et al. (2001) suggested a power–law relationship between drainage rate and liquid fraction; a comparison between this relationship and the later one suggested by Stevenson (2006) may be found in the appendix of Stevenson et al. (2009). Most recently, Lorenceau et al. (2009) have proposed drainage expressions that are entirely viscous.

As mentioned above, the fact that a log–log plot of wet-front velocity versus flowrate exhibits one slope or other is often taken as evidence of the efficacy of one drainage expression or another. In fact, that predictions of the rigid-walled foam drainage equation have been shown to under-estimate the actual drainage rate in all of the studies surveyed by Stevenson (2007a). In this paper, the following will be demonstrated:

1. The slope of the double-logarithmic plot referred to above can be a consequence of whether losses are predominantly viscous or inertial in nature.
2. Inertial losses can be significant in the drainage of foams that are either very wet or display relatively large Galileo number, even though the flow is wholly laminar, just as inertial losses can be significant for flow through a packed bed at relatively low Reynolds number. Such foams occur in high throughput foam fractionation columns described by Aguayo and Lemlich (1974), for instance and, as will be demonstrated in Section 6, in some conventional forced drainage experiments.

We will utilise computational fluid mechanics to demonstrate that losses at a sudden expansion of flow become significant at a channel Reynolds number within the Plateau borders of about 10.

The corollary of our observations is that a new drainage expression that takes into account viscous losses in the channels and inertial losses at the nodes is required. We will suggest that a form similar to that for the pressure gradient due to flow in a packed bed (Ergun, 1952) is appropriate. The proposed drainage expression is not completely mechanistic and requires the use of one adjustable constant.

At this juncture it is important to consider the work of previous workers who have invoked the Ergun equation (or variants thereon) to describe the hydrodynamic state of a wet pneumatic (i.e. continuously rising) froth. Langberg and Jameson (1992) suggested that a variant on the Ergun equation could be used to describe the very wet foam at the interface between the bubbly liquid and froth in a pneumatic foam, and fit data for a pneumatic foam, but preferred an approach based upon Richardson and Zaki's (1954) hindered settling correlation for simplicity. Bhole and Joshi (2007) took a similar approach and fitted a form of the Ergun equation to data for pneumatic flotation column froth. Apparently, those engaged in the study of pneumatic floatation froths have recognised the existence of the inertial loss term under certain conditions whereas this has tended to have been overlooked by those engaged in studies of liquid drainage through stationary foam. However, Vandenberghe et al. (2005) assumed losses in a pneumatic foam were entirely viscous. Ambulgekar et al. (2004) proposed a drainage expression based on expressions for flow through packed beds although their model was also entirely viscous and they investigated relatively dry foam. The present study is novel because it:

1. Demonstrates the conditions under which inertial losses in a foam become important.
2. Gives a theoretically derived viscous loss coefficient as well as a rationale for estimating the inertial loss coefficient, and
3. Demonstrates the efficacy of the approach against relevant forced drainage data.