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On the utilisation of vorticity and strain dynamics for improved analysis of stirred processes

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ABSTRACT

The present work is aimed at the development of methodologies suitable for the characterisation and quantification of the strain dynamics occurring in vortical structures in stirred vessels. Due to the complexity of mixing reactor design, a fundamental flow, such as that associated with a vortex ring, has been selected to develop a procedure to gain improved understanding of mixing mechanisms by assessing the competitive actions of the strain rate tensor, **S**, and vorticity vector, $\boldsymbol{\omega}$. This flow exhibits similarities with the trailing vortex structure around impeller blades and is employed as a paradigm to develop analytical approaches, which are then applied to the vortex structure produced around a Rushton impeller. The identification of stretching and compression regions of strain and the quantification of the relevant terms is discussed in the context of drop dispersion and with reference to previously reported observations.

Keywords: Mixing; Strain; Energy dissipation; Vortex ring; Trailing vortex

1. Introduction

Mixing is of paramount importance in the chemical, petrochemical and pharmaceutical industries, as almost half of their production lines involve mixing in stirred vessels, but further optimisation of related processes is hampered by incomplete understanding of the fluid mechanics of mixing. Consequently further investigations are necessary to obtain a better understanding of the different mixing mechanisms taking place at different spatial length scales in order to optimise chemical processes and reactor design. In the present work an effort is made to improve understanding of the strain fields of vortical structures in stirred vessels and particularly the trailing vortices, in the context of drop dispersion. In the mixing of a drop/blob of fluid of size l_b injected into the turbulent velocity field in a stirred vessel, the blob responds to the velocity field and deforms in response to the eddies that move within the field (Kresta and Brodkey, 2004). The response of the blob depends on the eddy size (l_e) and intensity. Encountering an eddy much larger than its own characteristic dimension ($l_e \gg l_b$), the blob is likely to be convected without much, if any, deformation. If $l_e = O(l_b)$, then the blob will be deformed. The kind of deformation encountered in this case should depend on the type of eddy structure encountered: e.g. if squeezed between two counter-rotating eddies as in between trailing vortices, the blob is most likely to be elongated. On the other hand, the action of two co-rotating eddies may break the blob into smaller globules. If $l_e \ll l_b$, the dispersion of the blob into many smaller ones is likely to occur, reaching eventually as the eddies become smaller, sizes of the order of the Kolmogorov scale and, under the action of diffusion, the Batchelor scale in due course. Of course, the deformation and break-up process described above is only indicative and is affected by interfacial tension and other parameters.

In the present work, quantification of the stretching/compression present in the vicinity of vortical structures and in particular the trailing vortices around impeller blades is considered to help shed light on the nature and magnitude of the deformations to which an element of fluid, or equivalently, a blob of a second fluid inserted into the bulk, will be subjected. This, together with knowledge of the interfacial tension involved, can help improve understanding of the effect of vortex dynamics on different liquids fed into the stirred vessel.

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Nomenclature

LDA	Laser Doppler Anemometry
2D	two-dimensional
Ω_{ij}	rate-of-rotation tensor (s^{-1})
ω_i	vorticity vector in the ith direction (s^{-1})
ω_i^*	vorticity in the ith principal direction of **S** (s^{-1})
ϵ	kinetic energy dissipation rate ($m^2\,s^{-3}$)
η	dynamic viscosity (Pa s)
ν	kinematic viscosity ($m^2\,s^{-1}$)
ϕ	angle from the leading blade (rad)
σ	viscous stress tensor (Nm^{-2})
θ	angle from the trailing blade (rad)
D	impeller diameter (m)
D_p	pipe diameter (m)
L	piston stroke (m)
l_e	eddy size (m)
l_b	blob size (m)
N	impeller speed (rpm)
r	radial coordinates (m)
Re	Reynolds number
S_{ij}	rate-of-strain tensor (s^{-1})
S_{ii}^*	rate-of-strain along the ith principal axis (s^{-1})
T	vessel diameter (m)
U_p	piston velocity ($m\,s^{-1}$)
U_{tr}	vortex ring translational velocity ($m\,s^{-1}$)
u_i	velocity in the ith direction ($m\,s^{-1}$)

Earlier works, such as those of Ali et al. (1981) and Chang et al. (1981), have considered drop dispersion mechanisms in vessels stirred by pitched-blade and radial discharge impellers, respectively. Two different dispersion mechanisms were identified, both occuring in the trailing vortex system emanating from the impeller blades: these were termed ligament stretching and turbulent fragmentation. These mechanisms are reminiscent of the $l_e = O(l_b)$ and $l_e \ll l_b$ situations mentioned earlier; although present for both types of impeller, they exhibited differences, while blade thickness was found to influence ligament stretching.

The methodologies presented in this paper can aid better understanding and quantification of deformation processes in stirred vessels, for example drop break-up of dispersed phases that occurs primarily in the vicinity of the agitator. Shear forces and turbulent fluctuations affect drop break-up; in relation to the observations made earlier, if the initial drop size is large compared to the turbulent scales, viscous shear has a major effect until the drop is broken up to smaller sizes, and subsequently turbulent fluctuations become dominant and shear effects become negligible. However, it should be noted that the methodology used to analyse the data is not restricted to large scale structures and laminar flow but can be also applied to turbulent flows in which case the turbulent fluctuating velocities would be taken into consideration (Davidson, 2004).

The trailing vortex and impeller stream region of a stirred vessel, on which the present study focusses upon, comprises a relatively small part of the volume of the vessel and, for the Rushton turbine case, contains only around 15% of the total kinetic energy, but it is where around 50% of the total turbulence energy is dissipated, while with other impeller designs the proportion of energy dissipated in this region has similarly been estimated at 45–60% of the total (Ng and Yianneskis, 2000). Similarly, the velocity gradients and resulting shear near the blades are much higher than those encountered anywhere else in the vessel and are expected to have a dominant effect on deformation of fluid elements/drops transported through the vessel.

Vortex dynamics and the quantification of the stretching/compression induced by vortical structures involves considerable complexity. For this reason, the relevant methodology is first developed on a comparatively simple flow, the vortex ring. The selection of this flow as a simple flow where the interaction of strain and rotation can be studied in detail was based on the observations of Davidson (2004), Southerland et al. (1991) and Meunier and Villermaux (2003). Such a flow exhibits similarities with the trailing vortices emanating from an impeller blade; it is however simpler and thus more amenable as a paradigm to help comprehend the far more complex trailing vortex flow.

The experimental configurations (vortex ring generator and stirred tank) in which the data analysed in his work were obtained are described in the following section, together with the respective experimental techniques employed for the data acquisition. Subsequently, the definitions of the deformation rate tensor and the quantities of interest to the present work are given in Section 3. The results are presented and discussed first in terms of the vortex ring and then of the trailing vortex strain dynamics in Sections 4 and 5, respectively.

The implications of the work and suggestions for further research are outlined in the final section of the paper.

2. Flow configuration and experimental apparatus

The phase-resolved Laser Doppler Anemometry (LDA) measurements of Schäfer (2001) were employed to calculate the vortex dynamics of interest for the trailing vortices. In Schäfer (2001), a standard vessel of diameter $T = 150\,mm$ stirred by a Rushton turbine of diameter $D = T/3$ and blade thickness $0.04D$ was used. The impeller clearance and speed were $T/3$ and $N = 2670\,rpm$ ($Re = ND^2\nu^{-1} = 2780$), respectively, while the working fluid was white oil, FC 2012 W, of viscosity $\nu = 4 \times 10^{-5}\,m^2\,s^{-1}$. A schematic diagram of a standard configuration tank is shown in Fig. 1. To minimise the refractive distortion at the tank cylindrical surface, the rig was equipped with a trough. A more extensive description of the vessel is given in Schäfer (2001) and Schäfer et al. (1996).

2D Particle Image Velocimetry (PIV) measurements were obtained to study the vorticity and strain dynamics of the vortex ring. The vortex ring flow was produced by a piston pushing a column of water inside a tube of diameter $D_p = 28.6\,mm$ which opened into an expansion chamber of water where the vortex ring was formed. A sketch of the experimental rig used is shown in Fig. 2.

The experiments for the vortex ring strain dynamics presented in this paper were carried for a piston stroke and velocity of $4D_p$ and $U_p = 7.5\,cm\,s^{-1}$, respectively. To prevent the formation of a trailing jet and piston vortices behind the vortex ring, the piston stroke was terminated inside the pipe at a distance of $1D_p$ from the tube exit. The flow field produced by the vortex ring was investigated in a region centered at an axial distance of $x \approx 7D_p$ from the pipe exit. The study was carried for $Re = 4290$ with the definition of the Reynolds number based on the slug-flow model, as indicated in Eq. (1) (Shariff