LONGITUDINAL AND TRANSVERSE DISPERSION IN POROUS MEDIA

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Abstract: In the present work, we have analysed the experimental data presented in literature to characterize dispersion in porous media, at different dispersion regimes. The vast amount of data obtained by our group, together with the extensive data available from other sources, mostly for air and water at room temperature, provide a very detailed representation of the functions $Pe_T = f_1\,(Pe_m, Sc)$ and $Pe_L = f_2\,(Pe_m, Sc)$. Empirical correlations are presented for the prediction of the dispersion coefficients (D_T and D_L) over the entire range of practical values of Schmidt number and Peclet number. The simple mathematical expressions represent the data available, in literature, with good accuracy and they are shown to be a significant improvement over previous correlations.

Keywords: longitudinal dispersion; transverse dispersion; porous media; diffusion; mass transfer; schmidt number.

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DOI: 10.1205/cherd07017

0263–8762/07/
$30.00 + 0.00

Chemical Engineering Research and Design

Trans IChemE,
Part A, September 2007

INTRODUCTION

The problem of solute dispersion during underground water movement has attracted interest from the early days of this century. Since the early experiments of Slichter (1905) and particularly since the analysis of dispersion during solute transport in capillary tubes, developed by Taylor (1953) and Aris (1956, 1959), a lot of work has been done on the description of the principles of solute transport in porous media of inert particles and in packed bed reactors (see Bear, 1972; Dullien, 1979).

When a fluid is flowing through a bed of inert particles, one observes the dispersion of the fluid in consequence of the combined effects of molecular diffusion and convection in the spaces between particles. Generally, the dispersion coefficient in longitudinal direction is superior to the dispersion coefficient in radial direction by a factor of 5, for values of Reynolds number larger than 10. For low values of the Reynolds number (say, $Re < 1$), the two dispersion coefficients are approximately the same and equal to effective molecular diffusion coefficient.

The detailed structure of a porous medium is greatly irregular and just some statistical-properties are known. An exact solution to characterize flowing fluid through one of these structures is basically impossible. However, by the method of volume or spatial averaging it is possible to obtain the transport equation for the average concentration of solute in a porous medium (Bear, 1972; Whitaker, 1967).

At a 'macroscopic' level, the quantitative treatment of dispersion is currently based on Fick's law, with the appropriate dispersion coefficients; cross stream dispersion is related to the transverse dispersion coefficient, D_T, whereas streamwise dispersion is related to the longitudinal dispersion coefficient, D_L.

Macroscopic modelling of dispersion processes in isotropic porous media is usually based on the convective-diffusion equation:

$$D_L \frac{\partial^2 C}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(D_T r \frac{\partial C}{\partial r}\right) = u\frac{\partial C}{\partial z} + \frac{\partial C}{\partial t} \qquad (1)$$

where C is the mean solute concentration, u ($=U/\varepsilon$, where U is the superficial velocity and ε the porosity of the porous media of inert particles with diameter d) the mean interstitial velocity of fluid and t the time.

A large number of theories, namely the theories based on a probabilistic approach, have been proposed to explain dispersion in porous media; however the theory of Saffman (1959, 1960), who modelled the microstructure of a porous media as a network of capillary tubes of random orientation, and Koch and Brady (1985) were the most referred.

STATE OF THE ART

Flow velocities and hydrodynamic dispersion coefficients are key parameters for description of fluid and solute transport in porous media. Dispersion topic has interested a vast scientific community, namely hydrology and contaminant studies, and for some time now it is treated at length in books on flow through porous media (e.g. Delleur, 1999; Fetter, 1999; Sahimi, 1995; Grathwohl, 1998; Bear and Verruijt, 1987; Marsily, 1986; Koch and Brady, 1985; Scheidegger, 1974; Bear, 1972; Fried and Combarnous, 1971).

Hydrodynamic dispersion includes both mechanical (convective) dispersion and molecular diffusion. For low fluid velocities values, namely in the limiting case where $u \to 0$, solute dispersion is determined by molecular diffusion ($D_L \cong D_T \cong D'_m$); for high values of u convection became dominates but the contribution of diffusion cannot be neglected.

The experimental dispersion data are most frequently presented in logarithmic plots of D_T/D_m (or D_L/D_m) versus Pe_m ($=ud/D_m$), spanning six or more orders of magnitude in D/D_m (or D_L/D_m). The result may give the impression of a narrow spread in the data [see Figures 1(a) and 2(a)], but that is only an illusion.

Indeed, if for gas flow the experimental data reported in literature are very concordant, in the case of liquid flow, at near ambient temperature, corresponding to values of Schmidt number, Sc, in the range $500 < Sc < 2000$, the data available are very 'disperse'.

For longitudinal dispersion, most of the experimental values of Pe_L ($=ud/D_L$) reported in the literature, for values of Sc in the range $500 < Sc < 2000$, are shown in Figure 1(b), and they form a 'thick cloud' at approximately, $0.3 < Pe_L < 2$. However, if the data in Figure 1(b) are cleared of the points obtained in columns that were either, too narrow ($D/d < 15$), in comparison with the size of the particles in the packing, or too short ($L/D < 10$), the picture in Figure 3 emerges.

For transverse dispersion, some considerations in Figure 2(b) were made. The data of Grane and Gardner (1961) present significant scatter about all data. Many of the points reported by Simpson (1962) were obtained in beds of sand bonded by resin and this will lead to the formation of particle agglomerates, with a resulting increase in values of D_T. Blackwell (1962) had already observed that in a bed of sand particles of 74 μm to 840 μm, values of D_T were considerably higher than expected as a result of particle agglomeration (which gives a larger 'apparent particle diameter'). At the lower end of Pe_m, only the points of Hiby (1962) seem to fall consistently above our data; however it is not to be excluded that Hiby data (at low Pe_m) are subject to wall effect, for the width of the test channel was only eight

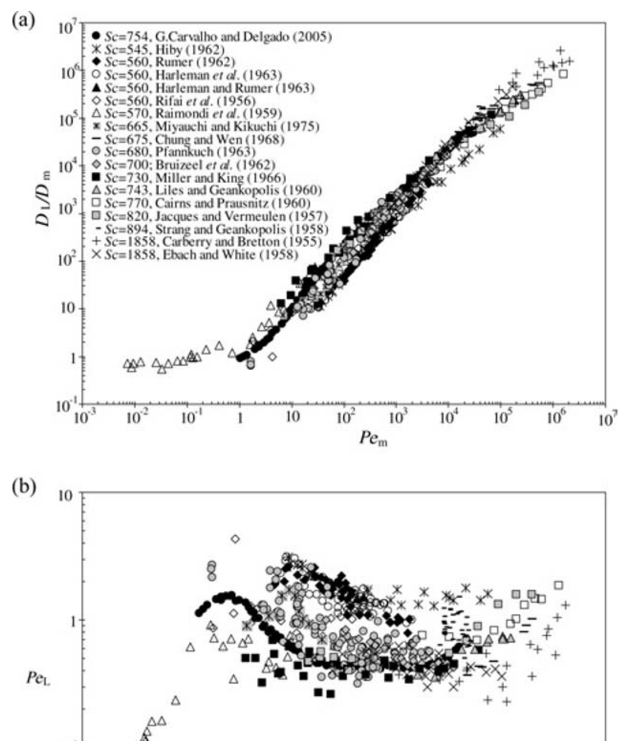

Figure 1. Longitudinal dispersion in porous media (a) D_L/D_m versus Pe_m and (b) Pe_L versus Pe_m.

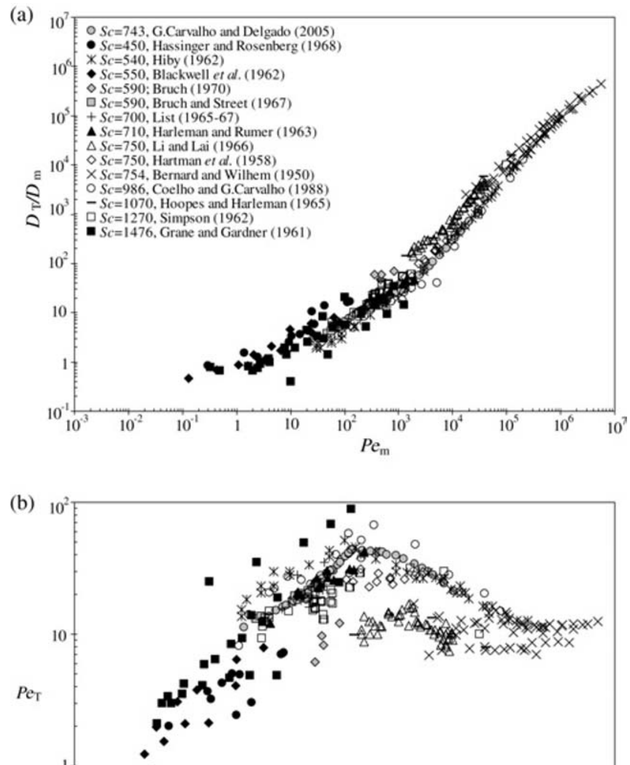

Figure 2. Transverse dispersion in porous media (a) D_T/D_m versus Pe_m and (b) Pe_T versus Pe_m.