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Research Report

Strategic differences in algebraic problem solving: Neuroanatomical correlates

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ABSTRACT

In this study, we built on previous neuroimaging studies of mathematical cognition and examined whether the same cognitive processes are engaged by two strategies used in algebraic problem solving. We focused on symbolic algebra, which uses alphanumeric equations to represent problems, and the model method, which uses pictorial representation. Eighteen adults, matched on academic proficiency and competency in the two methods, transformed algebraic word problems into equations or models, and validated presented solutions. Both strategies were associated with activation of areas linked to working memory and quantitative processing. These included the left frontal gyri, and bilateral activation of the intraparietal sulci. Contrasting the two strategies, the symbolic method activated the posterior superior parietal lobules and the precuneus. These findings suggest that the two strategies are effected using similar processes but impose different attentional demands.

1. Introduction

Mathematical cognition has been studied in a number of neuroimaging investigations. Although most studies have focused on the representation of numbers and on arithmetic computation (e.g., Dehaene et al., 2003; Menon et al., 2002; Rivera et al., 2005), there has been some recent attempts to study more complex mathematical operations. Anderson et al. (2003), for example, found algebraic transformation to be subserved by the left posterior parietal region and the left dorsal lateral prefrontal cortex. Sohn et al. (2004) found differences in prefrontal versus parietal engagement, depending on whether algebraic questions were presented in a verbal or symbolic format. In this study, we examined another aspect of algebraic problem solving: differences resulting from the use of different problem solving strategies.

In school, algebraic problems are often presented as stories or word problems (see Fig. 1 for an example). Like students elsewhere, students in Singapore often find these questions difficult. To give them better access, primary schoolers (10–12-year olds) are taught a diagrammatic or model method. Students are taught to draw diagrams, normally made up of rectangles, to represent relationships presented in word problems (see Fig. 1). The rectangles represent unknowns.

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Abbreviations: SE, Symbolic experimental; SC, Symbolic control; HIPS, Horizontal segment of the intra-parietal sulcus; ME, Model experimental; MC, Model control; PSPL, Posterior superior parietal lobules

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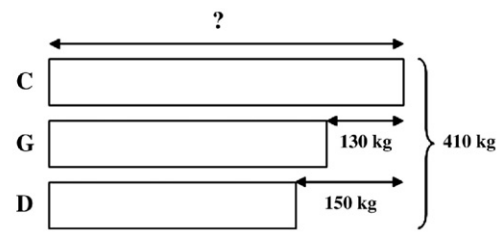

Fig. 1 – A model solution to the question: A cow (C) weighs 150 kg more than a dog (D). A goat (G) weighs 130 kg less than the cow. Altogether the three animals weigh 410 kg. What is the weight of the cow?

Students are expected to solve for the unknowns by analysing the quantitative relationships between the rectangles.

As students' success with word problems are affected by whether they understand the questions and whether they can transform the questions into equations or models (Carpenter et al., 1988; Kintsch and Greeno, 1985; Mayer, 1992; Riley and Greeno, 1988; Stacey and MacGregor, 1999; Verschaffel and De Corte, 1993), a strategy that requires explicit consideration of relevant relationships should promote accuracy. Indeed, Lewis (1989) showed that even college students benefited from training involving the use of pictorial representations.

The present study is part of an effort to examine whether the model method assists in the acquisition of formal or symbolic algebra (which, in Singapore, is taught in secondary or high school). Previous studies suggest teaching the model method may have both positive and negative effects. Findings from Khng and Lee (submitted) showed that even when instructed to use only symbolic algebra, students from secondary schools often exhibited intrusion errors and used the model method. Although such behaviour could be seen as adaptive in that students were using an alternative heuristic that was more accessible, many teachers saw the same behaviour in a negative light. In interviews and in feedback from in-service training, many secondary school teachers viewed the model method as childish, non-algebraic, and thought it a hindrance to the teaching of symbolic algebra (Ng et al., 2006).

A full answer to whether the model method assists in the acquisition of symbolic algebra will need to address cognitive, motivational, and pedagogical issues. In this study, we focused on the cognitive issues. We examined whether the model method and symbolic algebra were subserved by similar processes in adults with similar behavioural competency across the two methods. In terms of surface characteristics, the two methods seem to engage different types of information. The model method makes use of pictorial and alphanumeric information in depicting information. Symbolic algebra makes use of alphanumeric information only. Despite such differences, skilled mathematicians consider the two methods to be equivalent. The main difference being the way in which unknowns are represented: as rectangular boxes in the model method and as letters, x or y, in symbolic algebra. Because the model method has been part of the national curriculum in Singapore for over a decade, traditional programme evaluation techniques are of little assistance. In this study, we used functional magnetic resonance imaging (fMRI) to examine similarities and differences in processes that subserve the two strategies.

Information processing models of word problem solving guided the construction of experimental tasks. Most descriptions stipulated two stages: problem representation and problem solution (Bobrow, 1968; Briars and Larkin, 1984; Lewis, 1989; Riley and Greeno, 1988; cf. Koedinger and MacLaren, 2002). In a recent rendition, Mayer and Hegarty (1996) expanded these stages further. They argued that information such as quantitative relationships between protagonists is first extracted from the word problem. Pre-existing knowledge relevant to the problem is then activated and is integrated with the extracted information. Procedure

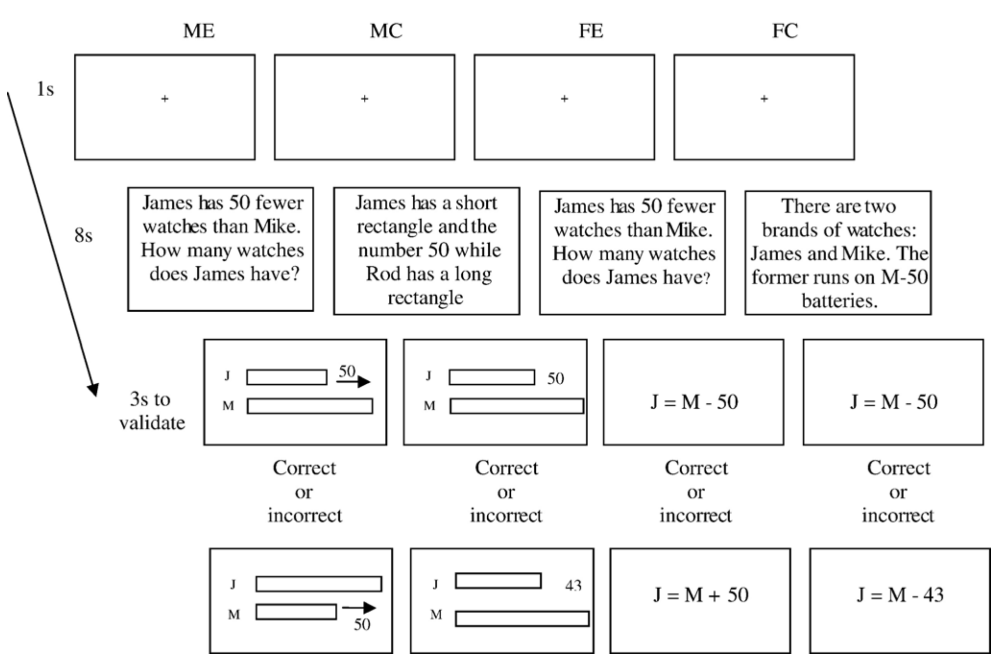

Fig. 2 – Experimental procedure.