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# Tensor decomposition of EEG signals: A brief review

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## HIGHLIGHTS

- EEG signals are naturally born with multi modes.
- EEG signals can be represented by the high-order multi-way array, tensor.
- Tensor of EEG can be exploited by tensor decomposition for multi-way analysis.

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## ABSTRACT

Electroencephalography (EEG) is one fundamental tool for functional brain imaging. EEG signals tend to be represented by a vector or a matrix to facilitate data processing and analysis with generally understood methodologies like time-series analysis, spectral analysis and matrix decomposition. Indeed, EEG signals are often naturally born with more than two modes of time and space, and they can be denoted by a multi-way array called as tensor. This review summarizes the current progress of tensor decomposition of EEG signals with three aspects. The first is about the existing modes and tensors of EEG signals. Second, two fundamental tensor decomposition models, canonical polyadic decomposition (CPD, it is also called parallel factor analysis-PARAFAC) and Tucker decomposition, are introduced and compared. Moreover, the applications of the two models for EEG signals are addressed. Particularly, the determination of the number of components for each mode is discussed. Finally, the N-way partial least square and higher-order partial least square are described for a potential trend to process and analyze brain signals of two modalities simultaneously.



**Contents**



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## 1. Introduction

Data processing and analysis plays a fundamental role in brain research using brain imaging tools. The recorded brain imaging data can be represented by a one-way series (called as a vector), a two-way array (called as a matrix), and a multi-way array (called as a tensor). Different signal processing and analysis methods are applied in terms of different ways of data representation.

### 1.1. One-way and two-way brain signals

For example, in the early stage of electroencephalography (EEG) studies, EEG data were represented by a time series and all data samples were carried by a vector (Berger, 1929). Since then, power spectrum analysis of the time series has been often applied for investigating EEG oscillations (Cohen, 2014; Niedermeyer and Lopes da Silva, 2005). Recently, time-frequency analysis (TFA) of the time series has been very attractive for analyzing single-trial EEG (Herrmann et al., 2013). Nowadays, multiple electrodes are often used to collect EEG data in the experiment. Therefore, EEG recordings naturally include two modes of time and space, at least. Subsequently, a matrix with the two modes has been extensively used to represent the EEG data. In some professional software for EEG data processing and analysis, EEG data are displayed on the screen (Delorme and Makeig, 2004). The horizontal axis of the plane (the row of a matrix) is for the time mode, and the vertical axis of the plane is for the space mode (the column of the matrix). As a result, the two-way signal processing methods including principal component analysis (PCA) and independent component analysis (ICA) have been performed on the matrix to remove artifacts and to extract brain activities of interest (Dien, 2012; Onton and Makeig, 2006; Vigario and Oja, 2008).

### 1.2. Multi-way nature of EEG signals

Indeed, in EEG experiments, usually there are more modes than the two modes of time and space. For instance, analysis of EEG signals may compare responses recorded in different subject groups (e.g. comparison of responses in a healthy control group and a clinical group). Thus, at least one more mode appears and it is the subject. Furthermore, in an experiment to elicit event-related potentials (ERPs), there are modes of EEG trial (since several stimulus presentations are required) and stimulus presentation condition. This means the brain data collected by EEG techniques can be naturally fit into a multi-way array including multiple modes.

However, the mostly applied computing tools for brain research are oriented for one-way or two-way data. Consequently, in order to facilitate the two-way signal processing methods, the extra modes besides the two modes of time and space are often concatenated (data are horizontally connected in a plane) or stacked (data are vertically connected in a plane) with the time or the space mode for generating a matrix (Calhoun and Adali, 2012; Cong et al., 2013b, 2014a; Delorme and Makeig, 2004; Dien, 2012; Eichele et al., 2011). This is often called unfolding a multi-way array into a matrix. For EEG data, such unfolding inevitably loses some potentially existing interactions between/among the folded modes, such as time, frequency and space modes. The interactions can be of research interest. Consequently, in order to appropriately reveal the interactions among multiple modes, the signal processing methods particularly for a multi-way array are naturally promising tools.

### 1.3. Multi-way array is a tensor, a new way to represent and analyze data

A multi-way array is named as a tensor (Cichocki et al., 2009; Kolda and Bader, 2009). For a matrix, matrix decomposition can be applied for data processing and analysis. Analogously, for a tensor, tensor decomposition can be applied as well. Tensor decomposition inherently exploits the interactions among multiple modes of the tensor. It was first defined in the field of mathematics (Hitchcock, 1927), and has been glorious in the fields of psychometrics and chemometrices for multi-way data analysis (Kroonenberg, 2008; Smilde et al., 2004). The existing key reviews for tensor decomposition often include its history, models, algorithms and various applications (Acar and Yener, 2009; Cichocki et al., 2015; Comon, 2014; Comon et al., 2009; Khoromskij, 2011; Kolda and Bader, 2009; Lu et al., 2011; Morup, 2011).

Recently, tensor decomposition has become surprisingly attractive for signal processing (Cichocki et al., 2015). Indeed, it has already been applied for analysis of ERPs in 1980s (Mocks, 1988a,b). In the past ten years, there have been many reports about tensor decomposition for processing and analyzing EEG signals. However, there is no review particularly for tensor decomposition of EEG signals yet. Therefore, this study is devoted to summarizing previous reports concerned with tensor decomposition of EEG signals and discussing the key issues regarding the application.

In the rest of the paper, next, tensors of EEG signals and tensor decomposition models are introduced. Subsequently, how tensor decomposition can be applied for processing and analyzing tensors of EEG signals is addressed. Finally, the potential trend for analyzing data of two modalities in brain research is stated. Since superiority