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Block-bootstrapping for noisy data

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HIGHLIGHTS

- For the first time, robust statistics is possible due to our new method.
- The proposed method outperforms conventional methods. It renders confidence intervals as small as possible.
- The new method precludes false positive conclusions.

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ABSTRACT

Background: Statistical inference of signals is key to understand fundamental processes in the neurosciences. It is essential to distinguish true from random effects. To this end, statistical concepts of confidence intervals, significance levels and hypothesis tests are employed. Bootstrap-based approaches complement the analytical approaches, replacing the latter whenever these are not possible.
New method: Block-bootstrap was introduced as an adaption of the ordinary bootstrap for serially correlated data. For block-bootstrap, the signals are cut into independent blocks, yielding independent samples. The key parameter for block-bootstrapping is the block length. In the presence of noise, naïve approaches to block-bootstrapping fail. Here, we present an approach based on block-bootstrapping which can cope even with high noise levels. This method naturally leads to an algorithm of block-bootstrapping that is immediately applicable to observed signals.
Results: While naïve block-bootstrapping easily results in a misestimation of the block length, and therefore in an over-estimation of the confidence bounds by 50%, our new approach provides an optimal determination of these, still keeping the coverage correct.
Comparison with existing methods: In several applications bootstrapping replaces analytical statistics. Block-bootstrapping is applied to serially correlated signals. Noise, ubiquitous in the neurosciences, is typically neglected. Our new approach not only explicitly includes the presence of (observational) noise in the statistics but also outperforms conventional methods and reduces the number of false-positive conclusions.
Conclusions: The presence of noise has impacts on statistical inference. Our ready-to-apply method enables a rigorous statistical assessment based on block-bootstrapping for noisy serially correlated data.

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1. Introduction

In the neurosciences, concepts, mechanisms, and characteristics often need to be inferred from measured signals. This typically renders powerful means for statistical evaluation necessary to preclude erroneous conclusions. For several data analysis approaches analytic evaluation schemes have been developed. For others, numerical Monte-Carlo based approaches are used to evaluate the statistical significance of the findings. The bootstrap has emerged as a powerful tool for this (Efron, 1979; Arndt et al., 1996; Davison and Hinkley, 1997; Foster and Bischof, 1987; Hentschke and Stüttgen, 2011; Zoubir and Boashash, 1998).

The general idea of bootstrap is to provide the distribution of a statistics based on the measured signals alone, when the analytic derivation of the statistics is not known. The distribution is sampled by randomly drawing with replacement from the measured data (Efron and Gong, 1983; Hall et al., 1995). Once this empirical distribution is obtained from the bootstrap, confidence intervals can be derived and hypothesis tests can be performed based on the empirical α-quantiles. Bootstrapping leads to a valid approximation of the true distribution of the test statistics under some assumptions (Mammen, 1992).

Among others, independence of the sampled data points is one assumption that has to be fulfilled in order to render bootstrapping sensible. When investigating time series as often measured in the neurosciences, this fundamental prerequisite for the applicability of bootstrap is violated. The temporal correlation, which characterises the dependence of the random variables of the time series, can be quantified by the autocorrelation function. This insight has led to the idea of block-bootstrapping (Carlstein, 1986; Hall et al., 1995; Künsch, 1989).

For temporally correlated data, block-bootstrapping draws with replacement from a set of independent blocks, i.e. snippets of the data (Davison and Hinkley, 1997). The appropriate choice of the block length is a key parameter and does not only depend on the measured time series but also on the analysis technique that is applied. "Optimality" in case of block-bootstrapping refers to the minimal squared distance between the true and the estimated quantity, yielding a trade-off between squared bias and variance (Peifer et al., 2005; Percival and Walden, 1993; Schelter et al., 2007).

The decay rate of the autocorrelation function as a measure of dependence in the data is the vital parameter in block-bootstrapping, as has been shown for the variance (Peifer et al., 2005) and mean phase coherence (Schelter et al., 2007), explicitly. It needs to be estimated as reliably as possible in order to render the segments as short as possible but long enough to guarantee independence. The decay rate in the autocorrelation can either be estimated by fitting an exponential function to the envelope of the empirical autocorrelation function or alternatively it can be estimated by modelling the process as an autoregressive one. While the latter is sensible only for small orders of the autoregressive model, the former provides a robust means for more complicated, potentially nonlinear dynamics as well.

A naïve choice of block length yields a bias or high variance of the statistics, and eventually fails in providing an optimal estimate for the block length. This is due to the influence of noise, becoming particularly important in the case of measurement noise or in cases in which the signal itself is modulated noise. While for the former the electroencephalography (EEG) is a prototypical example, the electromyography (EMG) is genuine for the latter.

As we demonstrate in this manuscript, the presence of these two types of noise strongly influences the determination of the optimal block length. A modification of conventional methods is necessary otherwise sub-optimal or even anti-conservative statistics can be obtained due to an underestimation of the block lengths. As we demonstrate here by both analytic calculations and simulations, the amount to which the length is underestimated is a function of the noise to signal ratio. We provide a modified block length selection approach that is robust with respect to the presence of a range of noise levels. We demonstrate the benefit of this new approach not only in model systems but also by investigating confidence intervals for the tremor amplitude based on EMG activity. The amplitude provides a measure for tremor severity and is therefore key to support physicians in the various tasks, such as the diagnosis or treatment of tremor. However, we emphasise that the proposed approach is not confined to EMG recordings and tremor data. Neither does it depend on recording modalities, such as EEG, EMG or fMRI, nor on scientific fields, such as tremor or epilepsy. Examples of its applicability are seizure detection as initiated by Gotman (1982), various studies concerned with network estimation, e.g. by the mean phase coherence (Schelter et al., 2007), or resting state studies (Bellec et al., 2010).

The manuscript is structured as follows. In Section 2 we present the EMG data of a tremor patient and the systems used to model them. We first (Section 2.1) introduce the EMG data that we aim to analyse and specify the impacts of noise onto the analysis. Based on the EMG data, parameters of the model system are adapted. As a second step (Section 2.2) we demonstrate the weaknesses of conventional block length selections in the presence of noise. We analytically derive how this can be overcome with our new robust block-bootstrapping. In Section 3 we apply block-bootstrapping to both the model system and the measured EMG signals of a tremor patient, deriving confidence intervals. We compare the confidence intervals obtained from the modified block length selection to the unmodified version, showing the superior efficiency of our method.

2. Material and methods

To motivate and illustrate the new approach to block-bootstrapping, we use an example of a representative recording of the wrist muscle activity of a tremor patient measured by EMG (see Fig. 1a). We model the EMG by autoregressive processes (Fig. 1b) in order to analytically show the effect of noise onto the block length selection.

2.1. Tremor example

Tremor is characterised by an involuntary oscillating movement of extremities. In essential tremor, a hereditarian form of pathological tremor, typically, the hands tremble at a frequency at around

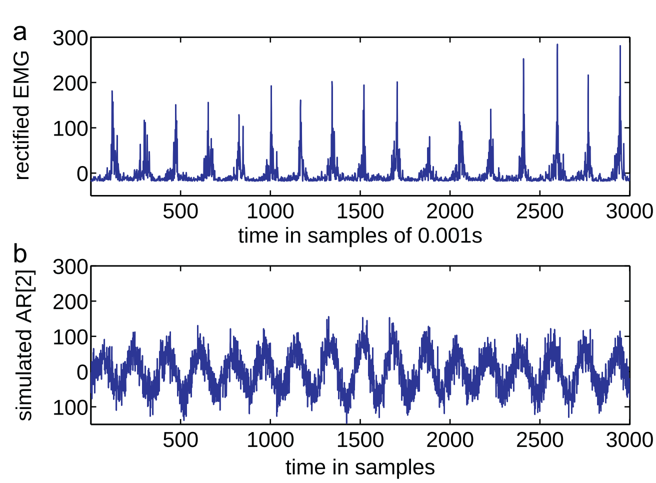

Fig. 1. Short section of data from a rectified EMG (a), and an autoregressive process (b) with parameters $a_1 = 1.9975$ and $a_2 = -0.9987$, intrinsic noise variance $\sigma^2 = 0.05^2$ and measurement noise variance $\Sigma^2 = 30^2$ (see Eqs. (1) and (2)).