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Calculating solar equivalence ratios of the four major heat-producing radiogenic isotopes in the Earth's crust and mantle

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ABSTRACT

As part of the ongoing work in defining a consistent and unified geobiosphere emergy baseline (GEB) this paper considers the radiogenic component of the available energy from geothermal sources (one third of the global tripartite: solar radiation, dissipation of tidal momentum, and geothermal exergy). Recent literature suggests that Earth's geothermal energy results from two very different sources, decay of radioisotopes and primordial heat (heat left from Earth's accretion). In previous baseline computations, the radiogenic component of geothermal exergy was added to primordial heat, given various names like "deep earth heat", and a single transformity was computed for the combined sources. With the acknowledgment that the geothermal component of the GEB had two different sources, it became apparent that a single transformity may no longer be appropriate, thus a method of computing separate transformities was necessary. In a novel approach, this paper uses gravity as the primary input to both solar radiation and heavy radionuclides and computes gravitational transformities for both. Then solar equivalence ratios (SERs) are computed between solar radiation and the four major crustal radionuclides (^{238}U, ^{235}U, ^{232}Th, ^{40}K). The SERs are combined with published radiogenic geothermal exergy data to calculate the solar equivalent exergy of the radiogenic component of the geothermal flux. This equivalence method can be used to derive a theoretically and methodologically consistent calculation for the other inputs to the global emergy baseline (i.e. tides and primordial geothermal heat flux) that can be similarly quantified in terms of gravitational exergy required to produce them.

1. Introduction

The ongoing work of defining a unified geobiosphere emergy baseline (GEB) has resulted in several key findings that are shaping our understanding of the sources, processes and flows of available energy that drive the geobiosphere. Early on, Raugei (2013) made two assertions that are fundamental to this current work. The first was that "...two exergy flows which were clearly produced by different processes (such as RadHeat (*radiogenic heat*) vs. CrustHeat (*crustal heat*) ...) should not be expected to have the same transformity..." The second in the same paper (Raugei, 2013) was "...since the origins of tidal exergy and 'deep earth heat' cannot be traced back to solar radiation, it is arguably conceptually impossible to compute solar transformities for them..." Raugei's first assertion resulted in our search for distinguishing radiogenic heat from primordial heat and then searching for methods of computing solar equivalences. The second assertion had profound effects on how that computation should be done, and ultimately on the terminology used to describe the results of that computation.

We must deal with the second assertion first in order to make sense of the terminology we will use to discuss the first assertion and for the rest of this paper. Raugei (2013) rightly observed that solar energy in no way actually contributes to radiogenic heat, primordial heat, or tidal dissipation. As a result, it is quite apparent that solar exergy is not embodied directly in any of these sources and it is inappropriate to characterize them as solar emergy or to characterize their equivalence ratio with solar exergy as a transformity (sej J^{-1}). These observations have lead to several clarifications in terminology (see Table 1). Since solar radiation is not directly 'embodied' in geothermal exergy, the ratio of geothermal exergy to solar radiation is not a transformity, but instead is a solar equivalence ratio (SER). Also, the result of multiplying a SER by the exergy of the geothermal source does not yield emergy, but instead, solar equivalent exergy. These are very important distinctions and are carried throughout this current work as well as the other papers in this special edition (Brown et al., 2016; Brown and Ulgiati, 2016a; Brown and Ulgiati, 2016b, Campbell, 2016; De Vilbiss et al., 2016).

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Table 1
Abbreviations used in this manuscript.

Abbreviation of symbol	Meaning
GEB	Geobiosphere emergy baseline
gej	gravitational emjoules
GPE	Gravitational potential energy
seJ	Solar equivalent joule
sej	Solar emjoule
SER	Solar equivalence ratio
$_{g}T_{S}$	Gravitational transformity of solar radiation
$_{g}T_{R}$	Gravitational transformity of radionuclide

As for Raugei's other assertion, while the heat released at Earth's surface is the result of the combined heat flows from primordial heat and radionuclide decay, these two sources of heat do not have the same properties and thus should not be expected to have the same SERs. Far from it, we agree with Raugei (2013), they should be expected to have different SERs. As a result of this supposition, we have undertaken this evaluation, using a novel approach to compute SERs for the four major radionuclides (^{238}U, ^{235}U, ^{232}Th, ^{40}K) responsible for a portion of the geothermal output of Earth.

1.1. *Forward vs. backward computation*

Our approach uses forward computation of the available energy required to synthesize the radionuclides. It is a departure from previous calculations of solar equivalence of the primary geobiosphere inputs as outlined by Odum (2000) and used by Brown and Ulgiati (2010, 2016a) as well as by Campbell (2016), which, in general, use backward calculation to establish an equivalence between the solar radiation and tidal dissipation and between solar radiation and geothermal flux. The difference between forward and backward computation is as follows. In a forward calculation one computes the available energy required to make something, in this case the energy required to synthesize the radionuclides. In backward calculation, one uses some form of equivalence between a given energy flux and another energy source, for instance, geothermal flux and solar radiation. While there are any number of ways this might be done, two common ways rely on either algebraic manipulation of equations (Odum, 2000; Brown and Ulgiati, 2010, 2016a) or through a third energy flux that can be related to both of the first two (Campbell, 2016).

In this study, we compute the gravitational exergy required to create the conditions for nuclear synthesis, a forward computation, and the gravitational exergy required to generate solar radiation, also a forward computation. Since both products (radionuclides and solar radiation) are the result of the dissipation of the same form of exergy (gravitational) a simple relationship can then be made between the ratios of gravitational exergy/radionuclide exergy and gravitational exergy/solar radiation, yielding an equivalence between solar radiation and radionuclide exergy, and generating a solar equivalence ratio for the latter.

1.2. *Production of light and matter*

The production of light and the synthesis of heavier forms of matter are simultaneous processes, and the resulting radiation and heavy elements are co-products. The energies involved in these reactions are well known. In our sun, production of solar radiation is primarily carried out by proton–proton chain reactions (fusion reactions) that combine hydrogen into helium. Nucleosynthesis of heavy radioactive elements does not occur in small stellar bodies like our sun, but instead primarily occurs in supernova stars many time more massive than the sun, catalyzed by gravitational collapse that releases the tremendous quantities of energy required to create the heaviest isotopes. In both cases, the driving force that creates the conditions for the production of light and radioisotopes is gravity. We assume equivalence between the gravitational exergy required to produce light in the sun and the gravitational exergy required for nucleosynthesis, albeit the latter requires much greater quantities.

2. Methods

2.1. *Background*

In this paper, we invoke some basic thermal and astrophysical concepts. Firstly, our emergy calculation relies heavily on the statistical mechanical concept of thermal energy, which uses a microscopic description of systems containing large number of particles. (Jacobs, 2013, p. 53) The various states of these particles give rise to whole system thermodynamic properties such as temperature, pressure, volume, and internal energy. Because of the assumption that particle numbers are large, we can treat the ensembles statistically and therefore use probability distributions to describe a system's state and derive its properties. (Jacobs, 2013, p. 53) The distribution we will use is the Boltzmann distribution as applied to an ideal gas (defined and explained in Jacobs (2013), pp. 62–70).

For the purposes of understanding stellar evolution, we use the statistical model of an ideal gas, which is based on the ideal assumptions of point-like (zero sized) particles with no intermolecular forces. (Blundell and Blundell, 2006, p. 8) This model relies on the concept of translational motion of particles, referring to each distinguishable particle's unique velocity given by its position and speed in all three spatial vectors. (Jacobs, 2013, p. 58) This leads naturally to the calculation of the translational kinetic energy of a particle as $1/2mv(x,y,z)^2$, and as we are concerned with the system as a whole, we will be using average translational kinetic energy of $1/2m\langle v^2 \rangle$.

We know now that the universe contains many galaxies in which stars are constantly being born through gravitational collapse leading to the condensation of gas in the interstellar medium. (Law and Rennie, 2015a) This collapse produces the high temperatures at which stellar nucleosynthesis takes place and from which the heavier elements and starlight are together forged. This gravitational collapse occurs only when the absolute value of the gravitational potential energy (E) in a large area is greater than the internal energy of the gas (U) itself:

$$|E| > U.$$

This happens at the so called Jeans density, which for hydrogen is about 5E−17 kg m^{-3}, (Ryan and Norton, 2010, p. 177) and the collapse proceeds until hydrostatic equilibrium is reached whereby the increased pressure on the inner surface of the star balances that created by the force of gravity.

We use thermal energy as a proxy for gravity, the reasoning for which, comes from the understanding that as the gravitational force condenses the particles comprising a star, it is doing work (W) equal to the potential energy (E) input from Gravity (Panat, 2008, p. 6):

$$dW = dE$$

This is negative work done in the process of an approximate adiabatic compression of an ideal gas, so that (Panat, 2008, p. 8):

$$dU = -dW = -P * dV, \quad \text{and} \quad -PdV = NK_B dT,$$