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Short communication

The vertical distribution of soil organic matter predicted by a simple continuous model of soil organic matter transformations

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ABSTRACT

New modification of the simple continuous model of soil organic matter (SOM) transformation is proposed. The modification allows to convert a SOM distribution over humification rate to a SOM distribution over depth and vice versa. Qualitative correspondence of calculated curve patterns of SOM vertical distributions to observed ones for different soil types is shown.

1. Introduction

Previously we proposed a mathematical model of SOM transformation based on using the rate of matter humification, h, as a continuous scale of its transformation degree (Bartsev and Pochekutov, 2015). This choice of the scale and description of SOM transformation processes in maximally generalized terms allowed making the model significantly simpler then other continuous models of SOM transformation (Ågren and Bosatta, 1998; Bosatta and Ågren, 2003; Bruun et al., 2010).

The model based evaluations of total SOM stocks in a number of different ecosystems coincide with those actually observed in these ecosystems. These evaluations were obtained at some assumptions about mineralization rate function and litter distribution providing analytical stationary solutions of the model equation.

Unfortunately the value h used as a scale is rather abstract and the direct measurement of SOM distribution over h is difficult. A possible method of its measuring is an examination of soil samples by means of a mass-spectrometer and correlation of more high-molecular compounds with lesser h-values. Therefore to provide the direct experimental verification and further practical use of the model it has to be modified.

The present study proposes a new modification of the model based on mapping the h-scale onto a depth within the thickness of soil profile. This mapping allows recalculating a SOM distribution over humification rate onto a SOM distribution over depth and vice versa.

2. Model equations

At the beginning let us reproduce in short the key formulae of the model (Bartsev and Pochekutov, 2015). The model is based on classical concepts of successive humification of SOM (Essington, 2004; Chertov and Komarov, 2013). For any organic compound in soil, the event rate of transformation of its molecules into more recalcitrant ones is considered as the rate of humification, h, specific to this compound. So, the multistage humification process is represented as a motion of SOM along the scale h from higher to lower h values.

The general model equation is:

$$\frac{\partial C(h,t)}{\partial t} - \frac{\partial}{\partial h}(v(h)\cdot C(h,t)) = -k(h)C(h,t) + D(h,t). \quad (1)$$

where $C(h, t)$ is a distribution of SOM over the rates of its humification h at time t. $D(h, t)$ is a distribution of plant litter supply per unit of time. Litter supply $D(h, t)$ can be given in carbon units, then $C(h, t)$ represents a distribution of soil organic carbon (SOC). $v(h)$ is a velocity of SOM motion along the h scale. $k(h)$ is SOM mineralization rate coefficient.

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As it was shown (Bartsev and Pochekutov, 2015), the choice of the humification rate coefficient, h, as a continuous scale of SOM transformation degree entails the following form of $v(h)$:

$$v(h) \equiv -\frac{dh}{dt} = h^2. \tag{2}$$

The theoretical evaluations of total SOM stocks were obtained from the stationary solutions of the Eq. (1) assuming two equations.

The first,

$$k(h) = bh^p, \tag{3}$$

is the simplest empirical nonlinear relation between h and k which provides an analytical solution of the model equation which is in good agreement to observed SOM stocks of several different ecosystems. Parameters b $(b>0)$ and p $(0<p<1)$ are certain empirical constants specific for specific environmental conditions. They are fitting parameters of the model at that b is an index of steepness of increase of k with increase of h, p is an index of nonlinearity of this increase.

The second,

$$D(h) = \sum_i D_{0i}\delta(h - h_{0i}), \tag{4}$$

where δ is Dirac delta function, index i numbers litter components, is the simplest description of litter supply as a sum of its components characterized by their initial humification rates h_{0i} and mean annual inputs D_{0i}. Values of h_{0i} are calculated from experimentally measured initial mineralisation rates k_{0i} for each i-th litter component, according (3), as $h_{0i} = (k_{0i}/b)^{1/p}$.

Since all organic compounds in the model are differentiated only by their h values then separate equation of the form (1) can be written for describing the distribution of transformation products of each i-th litter component – $C_i(h, t)$. The total SOM distribution is $C(h, t) = \sum_i C_i(h, t)$.

Assuming (3) and (4) steady-state solution of Eq. (1) for transformation products of each litter component takes the following form:

$$\bar{C}_i(h) = \frac{D_{0i}}{h^2} \exp\left(\frac{b}{p-1}(h^{p-1} - h_{0i}^{p-1})\right)(1 - \theta(h - h_{0i})), \tag{5}$$

where θ is the Heaviside step function.

Let us now consider the formation of the vertical SOM distribution in the soil profile only, ignoring processes in a litter layer over the soil surface. Then for the sake of simplicity let us assume that SOM motion in soil profile is unidirectional (downward).

In the course of humification the stability of SOM compounds increases, and it seems reasonable to suppose that the stability of their chemical bounds with soil mineral matrix increases too. Note that the relationship between how deep SOM is located in the soil and its stability and intensity of vertical transport was shown in (Nakane and Shinozaki, 1978). Therefore, it seems reasonable to assume that there is a relationship between the stability of SOM compounds (associated with h) and the vertical SOM transport velocity, w.

Following the pursuit of maximum simplicity we assume w is proportional to h:

$$w(h) \equiv \frac{dz}{dt} = ah. \tag{6}$$

Here z is depth in a soil profile and a is a certain coefficient which indicates a relation between SOM molecules recalcitrance to transformation and to transportation. The axis z is directed vertically downward, and the point $z=0$ is situated at the surface of the soil.

For the sake of simplicity, the plant litter is considered to be supplying only onto soil surface and characterized by an initial humification rate h_0. (Note, this assumption is fairly accurate for ecosystems in which the plant litter falling onto the surface prevails the root litter (e.g., forest ecosystems). For other ecosystems, this assumption may be used as a basic approximation showing the contribution of the surface litter components to the formation of SOM stock.) Then, the system of Eqs. (2) and (6) can be rewritten as the following equation

$$\frac{dh}{dz} = -\frac{h}{a} \tag{7}$$

and its solution, obtained at the condition $h(z=0) = h_0$ represents a one-to-one interrelation between h and z.

Two distributions of the same physical value over two different but one-to-one interrelated scales, such as in our case z and $h(z)$ must comply with the following equality:

$$\bar{C}(z)dz = -\bar{C}(h(z))dh. \tag{8}$$

It denotes that any micro-range of one scale and the correspondent micro-range of another scale must contain the same amount of matter. The "minus" sign here indicates that the direction from less to higher h-values corresponds to the direction from higher to less z-values and so the differentials dz and dh have an opposite sign.

Hence, the transformation of the distribution over one scale into the equivalent distribution over another scale is determined by the equation:

$$\bar{C}(z) = J \cdot \bar{C}(h(z)), \tag{9}$$

where $J \equiv -\frac{dh}{dz} = \frac{h}{a}$ is the Jacobian of transformation of the scale h to the scale z.

In the simplest case a can be considered as a positive constant. So we assume a linear relationship between velocity of SOM downward motion in the course of humification and its humification degree. Then the Eq. (7) has the following solution:

$$h(z) = h_0 \exp(-z/a). \tag{10}$$

This interrelation allows to transform SOM distribution over the scale h into SOM distribution over the scale z and vice versa.

Inserting Eq. (5) into (9) and expressing $h(z)$ in the form (10) gives the SOM distribution over depth z formed from one litter component:

$$\bar{C}(z) = \frac{D_0}{ah_0} \exp\left(\frac{z}{a}\right) \exp\left(\frac{bh_0^{p-1}}{p-1}\left(\exp\left(-\frac{z}{a}(p-1)\right) - 1\right)\right)\theta(z). \tag{11}$$

3. Patterns of curves of SOM distribution over soil depth

Let us consider vertical distributions of SOM formed from one litter component. Even with the extremely simple assumption about the relationship between w and h as the Eq. (6) with $a = const$ Eq. (11) demonstrates different patterns of vertical SOM distribution curve which are characteristic for a number of different soil types (Fig. 1).

Pattern 1α, a decaying curve is characteristic for cambisols, luvisols, ferralsols, acrisols (FAO classification, Zech and Hintermaier-Erhard, 2007). Pattern 1β, a decaying curve with a bend is characteristic for in umbrisols, gleysols, andosols (FAO, Zech and Hintermaier-Erhard, 2007). Pattern 1γ, an increasing first and then decaying curve is characteristic for brown earth (Glazovskaya, 1984).

The type of the pattern of distribution curve $\bar{C}(z)$ depends on the combination of the values of parameters b, p, and k_0. Details of this dependence can be shown by means of mathematical analysis. Parameter a only influences the stretching of the curve. The combinations of parameters p and b can be conveniently presented as