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Short communication

Capture the time when plants reach their maximum body size by using the beta sigmoid growth equation

Pei-Jian Shi[a,*], Lei Chen[b], Cang Hui[c,d], Henri D. Grissino-Mayer[e]

[a] *Collaborative Innovation Center of Sustainable Forestry in Southern China of Jiangsu Province, Bamboo Research Institute, Nanjing Forestry University, 159 Longpan Road, Xuanwu District, Nanjing 210037, China*
[b] *Graduate School of Environmental Science, Hokkaido University, N19W8, Sapporo 060-0819, Japan*
[c] *Centre for Invasion Biology, Department of Mathematical Sciences, Stellenbosch University, Matieland, South Africa*
[d] *Mathematical and Physical Biosciences, African Institute for Mathematical Sciences, Cape Town, South Africa*
[e] *Department of Geography, The University of Tennessee, Knoxville, TN 37996, USA*

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ABSTRACT

Of the many mathematical models proposed for capturing the dynamics of plant growth, the beta sigmoid function (BSF) is the newest and consequently is not well known to ecologists. A recent software package based on the Microsoft Excel macro, LEAF-E, was designed to promote the use of BSF, even though the performance of BSF and other growth models had not been compared. We developed R functions for fitting the BSF with a freer option for choosing the parametric number, and illustrated their performance using simulated data generated by four equations (the exponential, logistic, Gompertz, and von Bertalanffy equations), as well as dry weights of six crop species measured in growing seasons. Compared to other growth models, the BSF allowed for both symmetric and asymmetric growth curves, and thus the simulated data modeled the actual data quite well. It was demonstrated that the BSF was better than the above four traditional growth equations. In addition, the R functions developed here can facilitate future data fitting and model comparison for capturing plant growth dynamics. And the time when plants reach their maximum body size can be accurately obtained by using the BSF.

1. Introduction

Plant growth models are at the heart of plant metrics (Paine et al., 2012), where the dynamics of weight or height of focal plant species can be described (Shi et al., 2013). The growth of some plant tissues and organs can also be depicted by such models, such as the mean number of xylem cells, leaf length, and fruit size during the growing seasons (Huang et al., 2011; Voorend et al., 2014). Recently, a nonlinear regression-based tool, LEAF-E (Voorend et al., 2014), a macro within the Microsoft Excel platform, was devised for estimating the parameters of the beta sigmoid function (BSF), a model proposed by Auzanneau et al. (2011) for describing leaf growth, originally proposed by Yin et al. (2003). Here, we aimed to: (1) provide R functions for data fitting (R Core Team, 2015); (2) evaluate the performance of BSF using simulated data from four traditional growth equations (exponential, logistic, Gompertz and von Bertalanffy equations); and (3) evaluate the performance of BSF using actual plant growth data. In addition, we found that the BSF used by Voorend et al. (2014) could not be directly derived from that of Yin et al. (2003) as they had stated. Our equation was directly derived from the latter, and performed well both for simulated data and actual growth data of plants.

2. Materials and methods

2.1. Simulated and real datasets

We used the following growth equation to simulate plant growth data:

(i) Exponential equation

$$w = \begin{cases} w_0 \exp(rt) & \text{if } t < t_e \\ w_{\max} & \text{if } t \geq t_e \end{cases} \tag{1}$$

where w represents the weight at time t; w_0 is the initial weight at $t = 0$; $w_{\max}$ is the maximal weight at time $t = t_e$; and r is the instantaneous growth rate.

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* Corresponding author. Tel.: +86 25 85427231; fax: +86 25 85427487.
E-mail address: peijianshi@gmail.com (P.-J. Shi).

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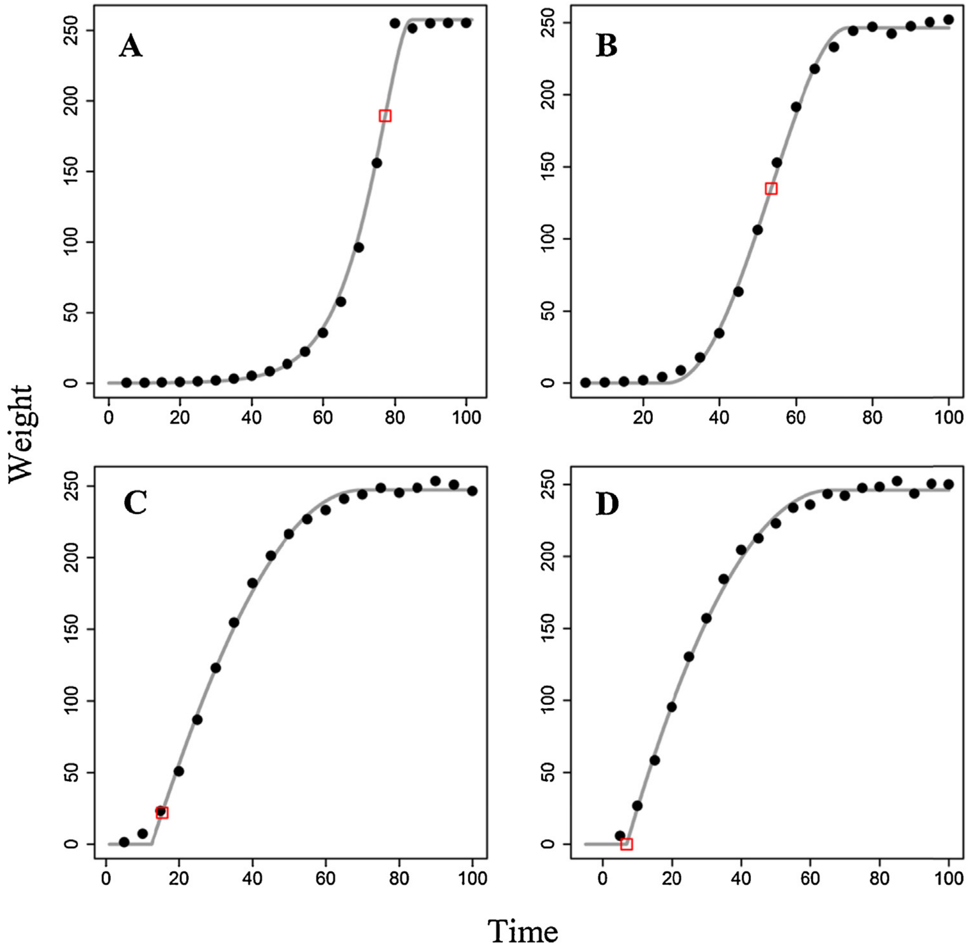

Fig. 1. Fitted results of the beta sigmoid function for simulated data. (A) Exponential equation; (B) logistic equation; (C) Gompertz equation; and (D) von Bertalanffy equation. The corresponding coefficients of determination are 0.9945, 0.9987, 0.9981, and 0.9971, respectively. The points represent the data simulated by the above growth equations with CV = 1% noise, and the gray curves are predicted values from the beta sigmoid function. Here, CV denotes the coefficient of variation.

(ii) Logistical equation

$$w = \frac{w_{\max}}{1 + ((w_{\max}/w_0) - 1)\exp(-rt)} \tag{2}$$

(iii) Gompertz equation

$$w = w_{\max} \exp\left[-\ln\left(\frac{w_{\max}}{w_0}\right) \cdot \exp(-rt)\right] \tag{3}$$

(iv) von Bertalanffy equation

$$w = w_{\max}\left\{1 - \left[1 - \left(\frac{w_0}{w_{\max}}\right)^{1/4}\right] \cdot \exp\left[\frac{-at}{4w_{\max}^{1/4}}\right]\right\}^4 \tag{4}$$

Here, a is a parameter. It is a special case of the generalized von Bertalanffy equation which is commonly referred to as the "ontogenetic growth model" (Shi et al., 2013).

We set $w_0 = 0.1$, and $w_{\max} = 250$ for all four equations during the simulations. For the exponential equation, $r = 0.098$, $t_e = 80$; for the logistic equation, $r = 0.15$; for the Gompertz equation, $r = 0.08$; for the von Bertalanffy equation, $a = 1.1$. These parameters were set just to make comparison apparent when $0 < t < 250$. In fact, we could change these parameters to other values and they will not affect the main results when using the BSF to fit the simulated data. The time was set to range from 5 to 250 with an increment of 5. To validate the model for data with noise, we added a normal random number with zero mean and 1% standard error to the simulated weights from the above models at each sampling point.

We chose six species of crops from the study by Shi et al. (2013). These crops were planted in field on 27 June, 2011. Measurements of total plant fresh and dry weights were performed on 15 dates

Table 1

Parametric estimates and the goodness-of-fit by using the beta sigmoid function to fit the dry weight of six crop species.

Common name	Latin name	c_m	t_m	t_e	RSS	χ^2	R^2
Kidney bean	*Phaseolus vulgaris* L.	0.34719	47.44	71.58	2.69	0.36	0.9910
Adzuki bean	*Vigna angularis* (Willd.) Ohwi et Ohashi	0.899	60.98	73.16	2.47	4.29	0.9981
Mungbean	*Vigna radiata* (L.) R. Wilczek	1.06537	67.64	84.31	11.26	0.92	0.9966
Cotton	*Gossypium spp.*	3.16052	71.84	85.22	120.53	17.90	0.9938
Sweet sorghum	*Sorghum bicolor* (L.) Moench	6.49894	66.33	80.09	516.39	10.17	0.9947
Corn	*Zea mays* L.	9.25776	70.73	89.42	1360.29	14.69	0.9946

Here, $t_b = 0$.