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An alternative surplus production model

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ABSTRACT

In this work we present a novel surplus production model for fisheries stock assessment. Our goal is to enhance parameter estimation and fitting speed. The model employs a production function that differs from the canonical logistic (Schaefer) and Gompertz (Fox) functions, but is still connected to the Pella–Tomlinson formulation. We embed this function in a state-space model, using observed catch-per-unit-effort indices and measures of fishing effort as input. From the literature we derive Bayesian prior densities for all model hyperparameters (carrying capacity, catchability, growth rate and error variance), as well as the state (annual stock biomass). We use the well-studied Namibian hake fishery as a case study, via which we compare the Schaefer, Fox and Pella–Tomlinson models with the new model. We also develop a package for the software R, which employs a Shiny application for data exploration, model specification, and output analyses. Posterior densities of hyperparameters and reference points agree across models. Identifiability issues emerge in the more cumbersome Pella–Tomlinson model. The new model yields small but consistent improvements in precision. It also renders implementation faster and easier, with no hidden truncation of negative biomasses. We conclude by discussing theoretical and practical extensions to this new model.

1. Introduction

Surplus production models provide simple descriptions of harvested populations, in terms of annual biomass levels (B_t), the intrinsic growth rate (r), the carrying capacity of the environment (K) and the efficiency of fishing gear (q; Hilborn and Walters, 1992; Polacheck et al., 1993). The basic concepts underlying these models were introduced by Graham (1935) and developed by Schaefer (1954), Beverton and Holt (1957), Pella and Tomlinson (1969) and Fox (1970). Albeit criticized for their potentially excessive simplicity (Megrey and Wespestad, 2013; Wang et al., 2014), surplus production models are still widely used today, to generate reference points for fisheries management, such as maximum sustainable yield (Hilborn, 2001; Zhang, 2013).

Several approaches have been employed to estimate parameters in surplus production models. Examples include ordinary least squares (Uhler, 1980), maximum likelihood (Gould and Pollock, 1997) and Bayesian inference (Walters and Ludwig, 1994). Some methods entail important assumptions, such as equilibrium, the existence of process and/or observation error, and prior information (Hilborn and Walters, 1992; Polacheck et al., 1993; Kuparinen et al., 2012).

Despite the parsimonious parameterisation of surplus production models, inference can be problematic. Often, the only information available stems from catch and effort data, which may not suffice for reliable inference (Hilborn and Walters, 1992; Xiao, 1998; Quinn and Deriso, 1999; Chen, 2003; Magnusson and Hilborn, 2007; Conn et al., 2010; Glaser et al., 2011; Cook, 2013). In light of this, it is important to examine posterior parameter correlation structure and uncertainty (Parent and Rivot, 2012), to avoid erroneous model output interpretation and mismanagement (Ludwig and Walters, 1981; Schnute and Richards, 2001; Needle, 2002; Wang et al., 2009; Conn et al., 2010; He et al., 2011).

When full estimation appears unfeasible, setting some model parameters to assumed fixed values is common in fisheries stock assessment modelling. However, several authors have found this to be poor practice (Rose and Cowan, 2003; Brooks et al., 2010; Brodziak and Ishimura, 2011; Lee et al., 2012). For instance, Mangel et al. (2013) showed that holding steepness and natural

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mortality constant fully determined key management reference points, whilst Gould et al. (1997) observed that ignoring variability in catch and effort data could overestimate the size of a fish stock by 20%.

Given the difficulties in estimation and the advice against setting too many constants, new estimation and modelling methods are still sought (Kuparinen et al., 2012). In this work, we propose a new surplus production model that facilitates parameter estimation. We establish a connection between the classical formulation Pella and Tomlinson (1969) and ours. After manipulation, we obtain a hierarchical multiplicative model, which can be linearised with respect to most parameters, via logarithmic transformation. To conduct Bayesian inference, we set up priors for all parameters. We describe how the model can be fitted to the well-studied Namibian hake fishery data set, and demonstrate benefits of the new model by comparing results with the Schaefer, Fox and Pella–Tomlinson surplus production models (Hilborn and Mangel, 1997; McAllister and Kirkwood, 1998; Parent and Rivot, 2012).

2. Methods

2.1. Data

We use catch (thousands of tons) and effort (thousands of hours trawled) data from the Namibian hake (*Merlucius capensis* and *M. paradoxus*) fishery, ICSEAF divisions 1.3 and 1.4, for the years 1965–1988 (ICSEAF, 1986; McAllister and Kirkwood, 1998). This fishery consisted of Spanish bottom trawlers in tonnage class 7 (1000-1999 GRT; Andrew, 1986).

2.2. Model

Bayesian state-space models typically consist of three layers (Berliner, 1996). The process layer characterises the temporal dynamics of a stochastic process, as a function of (time-invariant) hyperparameters. An observation layer connects this process with the observable variables. The third layer contains a description of the (prior) probability distribution of the hyperparameters and the state at the first time instant. In the sections below, we specify these three components, in the context of a surplus production model.

2.2.1. Standard process layer

In most situations, the total biomass (B) of an exploited population cannot be observed directly. Nevertheless, we may postulate a standard equation for its dynamics in discrete time t (Parent and Rivot, 2012), as

$$B_{t+1} = B_t + h(B_t) - C_t. \tag{1}$$

In this equation, C_t denotes total catch and $h(B_t)$ is a production function, that is, a parametric function that provides an estimate of biomass growth given its current level (Hilborn and Walters, 1992). In the classical approach of Pella and Tomlinson (1969), $h(B_t)$ is defined as

$$h(B_t) = \frac{r}{\phi} B_t \left(1 - \left(\frac{B_t}{K}\right)^{\phi}\right), \tag{2}$$

where r is the intrinsic rate of population growth, K is the carrying capacity of the environment and ϕ is a shape parameter. This leads to the production model

$$B_{t+1} = \left(1 + \frac{r}{\phi}\left(1 - \frac{B_t}{K}\right)^{\phi}\right) B_t - C_t. \tag{3}$$

With $\phi = 1$, the Schaefer (1954) production model is obtained:

$$B_{t+1} = \left(1 + r\left(1 - \frac{B_t}{K}\right)\right) B_t - C_t. \tag{4}$$

At the other extreme, the limit $\phi \to 0$ yields the Fox production model (Fox, 1970):

$$B_{t+1} = \left(1 + r\left(1 - \frac{\log B_t}{\log K}\right)\right) B_t - C_t. \tag{5}$$

From a theoretical standpoint, the addition of ϕ to the set of unknowns is sensible, as it allows the surplus production curve to be asymmetric in relation to stock size (Hilborn and Walters, 1992). However, many authors recommend fixing it, as fisheries data tend to be uninformative (Fletcher, 1978; Rivard and Bledsoe, 1978; Hilborn and Walters, 1992; Zhang, 2013).

The three process models described above can become stochastic, by multiplying the right hand side of the equations with $\exp[\epsilon_t]$, such that $\epsilon_t \sim N[0, \sigma]$, i.e. ϵ_t is i.i.d. Normal with mean zero and variance σ (Parent and Rivot, 2012).

2.2.2. Alternative process layer

While the Schaefer and Fox simplifications fix ϕ and keep r free, in this work we explore the opposite approach, which leads us to a more tractable equation for biomass dynamics. Specifically, we let the stock's intrinsic growth rate be a function of depletion ratio and shape parameter ϕ,

$$r_t = \phi \left(\frac{B_t}{K}\right)^{-\phi}, \tag{6}$$

yielding the production function

$$h(B_t) = \frac{r_t}{\phi} B_t \left(1 - \left(\frac{B_t}{K}\right)^{\phi}\right). \tag{7}$$

Hence, our approach also relinquishes one parameter (r) from the three available in the Pella–Tomlinson model; ϕ, on the other hand, is free but restricted to the interval (0,1).

Next, we point out that total catch, C, is often only partly observable, since it includes reported and unreported catches, as well as discards. Therefore, instead of C, we employ the fishing mortality rate F (also unobservable), such that

$$C_t = (1 - e^{-F_t}) \times (B_t + h(B_t)) \tag{8}$$

and

$$F_t \sim N[qE_t, \sigma]. \tag{9}$$

In Eq. (9), q is the (time-invariant and unknown) catchability parameter, and E is the (measurable) fishing effort. Randomness, with variance σ, may derive from transient fluctuations and possible long-term trends, unaccounted for fishing effort.

With Eqs. (6), (7) and (8), Eq. (1) simplifies to a product (Appendix A.1):

$$B_{t+1} = B_t^{1-\phi} K^{\phi} e^{-F_t}. \tag{10}$$

In Eq. (10), the consequences of extreme values of ϕ are worth studying. If $\phi = 1$, then biomass always bounces back to the carrying capacity, before the stock is harvested. In contrast, $\phi = 0$ leads inexorably to extinction, even for mild fishing mortalities. Based on these results, we call ϕ an elasticity parameter, and dub populations with high/low values of ϕ elastic/inelastic (see Appendix A.3 for illustration).

2.2.3. Reparameterization

To prepare the model for log-transformation, we reparameterize a few quantities in the process equation (10): define $\rho = \log(K)$ as the log-carrying capacity; let $\chi = \log(q)$ represent the log-catchability parameter; write $\beta_t = \log(B_t/K)$ as the log-transformed scaled biomass; and define the error term $\epsilon_t \sim N[0, \sigma]$.