Ecological Modelling 313 (2015) 181–200

Contents lists available at ScienceDirect

Ecological Modelling

journal homepage: www.elsevier.com/locate/ecolmodel

Emergy paths computation from interconnected energy system diagram

O. Le Corre[a,b,*], L. Truffet[a,c]

[a] Ecole des Mines de Nantes, La Chantrerie, 4 rue A. Kastler, BP 20722, 44307 Nantes Cedex 3, France
[b] Dpt. Systèmes Energétiques et Environnement, France
[c] Dpt. Automatique-Productique, France

ARTICLE INFO

Article history:
Received 10 March 2015
Received in revised form 17 June 2015
Accepted 18 June 2015

Keywords:
Memory algebra
Emergy algebra
Path-finding problem
Rewriting system
Thue system

ABSTRACT

Emergy is becoming an important sustainability ratio. Emergy analysis of real systems cannot be summarized to the trivial analysis of systems with several sources and one product. In such case only the application of the first rule of emergy algebra (all sources of emergy to a process are assigned to the process output) is required and the result can be stored in a simple table.

But real systems of interest (ecosystems, complex chemical production systems, etc) are interconnected systems with feedbacks, splits and by-products. As a consequence mathematical foundations for exact emergy analysis appear to be necessary. Indeed, emergy computation within an energy system diagram is based on four rules, the use of which is sometimes confusing. The complexity of the computation of emergy within interconnected system comes also from the fact that it does not obey Kirchoff-like circuit law. Thus, linear algebra approach fails.

Emergy computation follows a logic of memorization, thus the evaluation principles deal with pathways. An emergy pathway is a pathway from a source of emergy to a product which represents the sequel of assignations of the emergy source by the processes of the interconnected system.

Looking at pathways in ecosystems analysis is not a new subject. Because of the rules of emergy computation (specifically the no double counting rule) among an abundant classification of pathways in ecosystems the emergy pathways appear to be particular elements of only two classes of pathways: the class of simple pathways and the class of terminal non-feedback cycle pathways.

This paper is the companion paper of a previous paper written by the same authors. In the previous paper it is explained how to compute emergy flowing on a given arc of an emergy graph assuming all emergy pathways ending by this arc known. As explained in this paper an emergy graph is a transformed graph from an energy system diagram.

The aims of this paper are as follows. (i) Explain how to transform an energy system diagram into an emergy graph. (ii) Provide a clear and new algorithm easily programmable to compute emergy pathways. (iii) Insert this new algorithm in the whole procedure for computing an emergy flowing between two processes. (iv) Apply the whole procedure on a real world example. This example shows that the emergy content from meadowland-sun can either carry out by the milk or by the electricity (when considering the biogas and the CHP plant). In the general case, this emergy amount is lesser than the total meadowland-sun emergy.

1. Introduction

Odum (1996) has defined the emergy as the total available energy/exergy of one form that was used up directly or indirectly in the work of making a product or a service. Scienceman (1989) has coined emergy as “energy memory”.

Emergy analysis or emergy calculus within systems with few interconnected processes (typically several independent emergy sources and one output) can easily be solved without sophisticated mathematical apparatus. However, systems of practical interest have many interconnected processes, splits, feedbacks and by-products (see e.g. Mu et al., 2012).

Emergy is a concept. It means that emergy analysis cannot be validated by experimentation except for some trivial cases. Thus, as in e.g. theoretical physics emergy analysis of a given

* Corresponding author at: Ecole des Mines de Nantes, La Chantrerie, 4 rue A. Kastler, BP 20722, 44307 Nantes Cedex 3, France. Tel.: +33 251858257; fax: +33 251858299.
E-mail address: Olivier.Le-Corre@emn.fr (O. Le Corre).
URL: http://www.emn.fr/truffet (L. Truffet).

http://dx.doi.org/10.1016/j.ecolmodel.2015.06.030
0304-3800/

system requires at least a rigorous mathematical framework. This mathematical framework is represented by a formula or a set of axioms/rules which allows us to compute the emergy. The complete axiomatization of emergy calculus is defined in Le Corre and Truffet (2012b). For systems with only splits and feedbacks Tennenbaum (1988) proposed an exact formula. For systems with splits, feedbacks and by-products it seems that Brown in Brown and Herendeen (1996) was the first to propose the following four rules of emergy computation named *emergy algebra*. They are also explained in Odum (1996, Chapter 6).

(R1). All source emergy to a process is assigned to the processes's output(s).
(R2). By-products from a process have the total emergy assigned to each pathway.
(R3). When a pathway splits, the emergy is assigned to each 'leg' of the split based on its percent of total available energy flow on the pathway.
(R4). Emergy within a system of interconnected components cannot be counted twice.
(R4.1). Emergy in feedbacks cannot be double counted;
(R4.2). By-products, when reunited, cannot be added to equal a sum greater than the source emergy from which they were derived.

A consequence of emergy algebra is that emergy calculus does not obey Kirchoff-like circuit law. Hence, all simple approaches of emergy analysis of systems with splits, feedbacks and by-products based on linear algebra cannot be exact. Recently, it has been proved in Le Corre et al. (2015) that the approach based on ordinary (i.e. linear) algebra developed by Tennenbaum (2014) could not be exact for simple models with splits and feedbacks. More general models with by-products have to be studied in future work.

But the major drawback of the rules of emergy algebra is that at each step of the emergy analysis of a system a computer machine cannot 'decide' which rule has to be applied.

Thus, before elaborating any program such as e.g. SCALE (see Marvuglia et al., 2013) emergy analysis requires clear algorithms (or closed formulas) for calculating emergy flowing between two components of a system. To obtain algorithms/closed formulas a set of consistent axioms (or rules) is needed.

Since Brown and Herendeen (1996) only few methods not based on linear algebra have been proposed. In Li et al. (2010) the method is based on a preconditioning summarized into seven cases. But approximation is made when studied the output emergy at node G of graph (see their Figures 8 and 9, the remark in Lazzaretto, 2009, p. 2201 and the remark 4.2 in Le Corre and Truffet, 2012a). In Bastianoni et al. (2011) ingenious set theory is introduced. But, for systems with split and feedback the computation of emergy set X requires to solve a set equation of the form (see Bastianoni et al., 2011, Figure 5):

$$X = A \cup f(X),$$

where A is a known set and f is a set function. The authors do not indicate a clear procedure to solve it. As the authors notice, in Kazanci et al. (2012) the three elaborated methods do not treat the problem of by-products. In Mu et al. (2012), as in this paper, authors note that the rules (R1)–(R4) are "incomplete, hindering their use in the analysis of complex systems". They develop a method named *virtual emergy* which respects rule (R1).

The main remark is that all these methods 'forget' that the *Track summing method* developed by Odum and Tennenbaum (see Tennenbaum, 1988; Odum, 1996) is based on the computation of the emergy pathways from a source to the output of a node of an energy system diagram at which the emergy has to be calculated. In Le Corre and Truffet (2012a) a method based on a reinterpretation of the rules of emergy algebra into consistent axioms is developed. This method is validated on examples certified by Brown, Odum and Tennenbaum. This method assumes that a graph named *emergy graph* constructed from an energy system diagram is given. The knowledge of the emergy graph implies in particular that all emergy pathways are known.

Pathways or paths in (embodied) energy flow analysis of (eco)systems play also an important role. It is possible to relate energy analysis of interconnected systems with the computation of a matrix of the form:

$$\boldsymbol{\Lambda}(s) = (s\boldsymbol{I} - \boldsymbol{Z})^{-1}, \quad (1)$$

with s a real number, $\boldsymbol{I}$ denotes the identity matrix. The matrix $\boldsymbol{\Lambda}(s)$ is the *resolvent matrix* of the matrix $\boldsymbol{Z}$. Resolvent matrix appears in different contexts.

- In control theory when computing the transfer function, i.e. the ratio between the Laplace Transform of the inputs of a system by the Laplace Transform of the outputs of the system (see e.g. Astrom and Murray, 2008; Sontag, 1998).
- In the study of Markov processes (see e.g. Kallenberg, 2002).

For the particular case $s = 1$, $\boldsymbol{\Lambda}(1)$ is known as the Green function. Such a matrix naturally appears in several domains.

- In potential theory of Markov chains (see e.g. Revuz, 1984).
- In the study of the discretized heat equation (see e.g. Doob, 1959).
- In economics where this matrix is also known as Leontief/Ghosh inverse (see e.g. Leontief, 1973; Oosterhaven, 1996).

The matrix $\boldsymbol{\Lambda}(1)$ can be expanded as:

$$\boldsymbol{\Lambda}(1) = \boldsymbol{I} + \boldsymbol{Z} + \boldsymbol{Z}^2 + \cdots + \boldsymbol{Z}^n + \cdots. \quad (2)$$

Usually, $\boldsymbol{Z}^n(i, j)$ corresponds to a certain quantity which is related to paths of length n from node i to node j in the interconnected system modeled by a graph. As a basic example, let $\boldsymbol{Z}$ be the $\{0, 1\}$-labeled adjacency matrix of the graph associated with the interconnected system, that is $\boldsymbol{Z}(i, j) = 1$ if there is an arc between subsystem i and subsystem j, 0 otherwise. Then, $\boldsymbol{Z}^n(i, j)$ is the number of paths of length n and $\boldsymbol{\Lambda}(1)(i, j)$ denotes the number of all paths from node i to node j (see e.g. Patten, 1985; Ulanowicz, 1986; Fath and Halnes, 2007).

As in emergy analysis the enumeration of paths is also of practical importance for ecosystems. There exist different methods to enumerate paths (see e.g. Whipple, 1998, 1999; Patten, 1985; Patten and Higashi, 1995; Ulanowicz, 1983, 1986 and references therein). It is possible to choose one of these methods and eliminate all paths which are not emergy paths (see Section 2 for the precise definition of an emergy path). Another possibility is to use the *track summing* method developed by Tennenbaum (1988) or a variant based on graph search theory as in Marvuglia et al. (2013). But in this paper another method is chosen. To the best knowledge of the authors this method seems to be a new approach in ecosystems analysis literature. First of all, let us remark that the enumeration of all paths in a graph can be addressed as follows. Let us consider an ecological network modeled by a graph. Let $\mathcal{A}$ be the set of the arcs of the graph. Define the $\mathcal{A}$-labeled adjacency matrix of the graph $\boldsymbol{A}$ by $\boldsymbol{A}(i, j) = [i\,; j]$ if $[i\,; j]$ is an arc of the graph, $\underline{0}$ otherwise (the exact significance of the null word $\underline{0}$ is explained in Section 2). Then, enumerating all the paths of the graph is equivalent to the computation of the matrix:

$$\boldsymbol{A}^+ = \boldsymbol{A} \cup \boldsymbol{A}^2 \cup \cdots \cup \boldsymbol{A}^n \cup \cdots, \quad (3)$$