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Quantum structure in competing lizard communities

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ABSTRACT

Almost two decades of research on applications of the mathematical formalism of quantum theory as a modeling tool in domains different from the micro-world has given rise to many successful applications in situations related to human behavior and thought, more specifically in cognitive processes of decision-making and the ways concepts are combined into sentences. In this article, we extend this approach to animal behavior, showing that an analysis of an interactive situation involving a mating competition between certain lizard morphs allows to identify a quantum theoretic structure. More in particular, we show that when this lizard competition is analyzed structurally in the light of a compound entity consisting of subentities, the contextuality provided by the presence of an underlying rock-paper-scissors cyclic dynamics leads to a violation of Bell's inequality, which means it is of a non-classical type. We work out an explicit quantum-mechanical representation in Hilbert space for the lizard situation and show that it faithfully models a set of experimental data collected on three throat-colored morphs of a specific lizard species. Furthermore, we investigate the Hilbert space modeling, and show that the states describing the lizard competitions contain entanglement for each one of the considered confrontations of lizards with different competing strategies, which renders it no longer possible to interpret these states of the competing lizards as compositions of states of the individual lizards.

1. Introduction

This article looks into the challenging question of whether quantum structures are present in aspects of animal behavior. More specifically, we discuss an example that reveals contextuality and the appearance of entanglement in a proposed quantum theoretic model for the mating competition of three male morphs of *Uta stansburiana* lizards.

The first step leading to the result we put forward in the present article was related to our study of biological evolution based on a specific situation involving the rock-paper-scissors (RPS) game as an example. We observed (Aerts et al., 2011) that when the RPS game was regarded as a coincidence experiment, it allowed for violation of Bell's inequality (Bell, 1964).

The presence of contextuality in a situation of a compound entity consisting of two subentities essentially means that what happens with one of the subentities affects the behavior of the other subentity, which as a general situation is quite common. Contextuality can hence readily be identified in the case of the RPS dynamics, when the two players involved in the interaction are looked upon as a compound entity comprising two subentities. Indeed, whether one of the players wins or loses depends essentially on what the other player does.

It has been shown in the foundations of quantum theory that if this contextuality – in addition to its readily identifiable effect of one subentity functioning as a context for the other subentity – leads to a violation of Bell's inequality, this is indicative of the presence of a special type of contextuality which cannot be modeled classically and which, when modeled quantum-mechanically, is expressed by the appearance of entanglement in the state of the compound entity (Accardi and Fedullo, 1982; Aerts, 1986; Pitowsky, 1989). In the following we will refer to this type of 'Bell's inequality violating contextuality' as 'non-classical contextuality'. The presence of entanglement in the state of the compound

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entity means that this state cannot be described any longer as the composition of two states, where each one is a state of one of the subentities. The compound entity entails a new type of difficulty when attempted to be interpreted as two subentities, which is the fingerprint of the presence of quantum structure for any situation of compoundness. This is why finding Bell's inequality to be violated by the ideal RPS game to us was a straightforward reason to investigate whether a quantum structure was involved in this RPS dynamics.

The relevance of this insight to biology only became clear when some of us learned that the RPS game had been used as a modeling scheme for specific types of dynamical situations in population ecology, including the famous 'paradox of the plankton', where it is referred to as cyclic or multiple competition (Huisman and Weissing, 1999; Huisman et al., 2001; Schippers et al., 2001; Laird and Schamp, 2008; Allesina and Levine, 2011). Also, situations of competing lizard species were studied intensively by considering cyclic competition as a fundamental aspect of their dynamics. More specifically, one of us discovered an RPS strategy in the mating behavior of the side-blotched lizard species *Uta stansburiana* (Smith, 1996; Sinervo and Lively, 1996). It was found that males, having either orange, blue or yellow throats, follow heritable mating strategies. As in the RPS game, where scissors cut paper, rock crushes scissors, and paper wraps rock, the three-morph mating system is such that the wide-ranging ultradominant strategy of orange males is defeated by the sneaker strategy of yellow males, which is in turn defeated by the mate-guarding strategy of blue males. The orange strategy defeats the blue strategy, to complete the dynamic cycle. This 'lizard game' presents a stable pattern in the replicator dynamics where the dynamical system follows closed orbits around a mixed strategy Nash equilibrium (Smith, 1996; Sinervo and Lively, 1996; Sinervo, 2001; Sinervo et al., 2006, 2007). And indeed, if we regard two competing lizard morphs as a compound entity of two individual lizard morphs, we can recognize the same type of contextuality that we identified for the ideal RPS game; whether one of the lizards in competition will impregnate a female depends essentially on the color of the other lizard. Additionally, Bell's inequality is violated also for the lizard morphs, as we will explicitly show in Section 4, investigating in detail the contextuality that is apparent in the lizard competition. This explains our motivation to build a quantum-theoretic model for this lizard ecosystem.

Identification of quantum structure in the lizard dynamics can be seen as an example of the use of the mathematical formalism of quantum theory as a modeling instrument in domains different from the micro-world. This approach has led to interesting results in recent years and is now an active and emergent research field in itself. In cognitive science (concept theory and decision theory), in economics (finance and behavioral economics), and in computer science (semantic theories, information retrieval, and artificial intelligence), several situations have been identified where application of classical structures is problematic, whereas modeling based on quantum structures is successful (Aerts and Aerts, 1995; Van Rijsbergen, 2004; Aerts and Czachor, 2004; Aerts and Gabora, 2005; Busemeyer et al., 2006, 2011; Pothos and Busemeyer, 2009, 2013; Bruza et al., 2009; Aerts, 2009; Lambert-Mogiliansky et al., 2009; Khrennikov, 2010; Trueblood and Busemeyer, 2011; Aerts et al., 2013b,a; Busemeyer and Bruza, 2012).

An important point to be made for the above-mentioned approaches is that it is not the presence of microscopic quantum processes that is considered to be at work to give rise to the appearance of quantum structure in these different domains. Rather the situation is such, that it is possible to identify in these domains some typical quantum features, such as the quantum-type of contextuality and entanglement, and it are these features themselves that give rise to the presence of quantum structure. We will identify this type of quantum structure for the lizard ecosystem. It is interesting to mention in this respect, that in a comparable way such quantum structure has been found to be quite systematically present in human cognition, in the processes of decision-making (Aerts and Aerts, 1995; Busemeyer et al., 2006, 2011; Pothos and Busemeyer, 2009, 2013; Trueblood and Busemeyer, 2011; Busemeyer and Bruza, 2012), and in the dynamics of how humans use and combine concepts (Aerts and Gabora, 2005; Aerts, 2009; Aerts et al., 2013b,a).

In our investigation of the lizard ecosystem we construct an explicit quantum-theoretic representation in a complex Hilbert space of the underlying RPS-like dynamics that gives rise to the cyclic pattern of frequencies in the population identified experimentally. To accomplish this, we make use of the specific rules of the quantum formalism to calculate the probabilities in this underlying RPS-like dynamics in a way that allows to faithfully represent the experimental data gathered by one of us on the population frequencies over the last two decades. In Sections 2 and 3, we introduce our lizard system and explain the main aspects of our approach and modeling of the underlying RPS-like dynamics. We analyze how contextuality is one of its essential features. The latter notion is analyzed in detail with respect to the lizard competition in Section 4. In Section 5, we put forward the notions of the quantum-mechanical formalism that are needed in our paper, and in parallel we work out a Hilbert space model for the RPS-type lizard game. We show that the self-adjoint operators representing the confrontation events (called 'measurements' in quantum jargon) in the lizard competition do not commute, which means that the probability structure connected to them is non-classical. In Section 6 we analyze the 'lizard morphs situation' explicitly from the perspective of a compound entity consisting of a subentities situation, a situation well-known and studied in quantum theory, and we show that, following such a quantum analysis of compoundness, this lizard morphs situation involves entanglement in its states for each of the considered measurements. The problem with a Kolmogorovian probability model for the lizard game is analyzed in Section 7. Finally, in our conclusions of Section 8, we put forward ideas for future investigation. The general result obtained supports intuitions that dynamical systems based on non-Kolmogorovian probability may provide a fruitful conceptual framework for real-life interactions of populations (Aerts et al., 2013).

2. The RPS-type nature of the lizard dynamics

Before we provide proof of a quantum-like dynamical structure underlying the competing morphs of the lizard *Uta stansburiana*, we briefly sketch some game-theoretic aspects of population ecology.

Species competition can be reformulated in terms of evolutionary game dynamics describing how the frequencies of strategies within a population change in time, according to their success. Game theory typically deals with an individual (*player*) who is engaged in a given interaction (*game*) with other players and can decide between different options (*strategies*). Depending on the strategies of a player and its co-players a *payoff* is realized, and the possible maximization of this payoff is one of the fundamental aspects of game-theory. Evolutionary game dynamics thus deals with populations of players programmed – genetically or possibly also induced by the environment – to use the same strategy. Strategies with high payoff will spread within the population, where the payoffs depend on the actions of the co-players and hence on the frequencies of the strategies within the population. In classical evolutionary game theory, one typically assumes that the elements of the pay-off matrix are time invariant and evolution of the system takes place as frequency-dependent fitness changes, thereby changing the relative success and the probability