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A simple bistable model for reforestation in semi-arid zones, or how to turn a wasteland into a forest

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ABSTRACT

The advance of deforestation worldwide necessitates the development of models that facilitate reforestation, especially in areas where access to water is limited. To this end we have developed a model based on two ODEs, one for soil moisture (loamy sand type) and the other for plant biomass of *Pinus halepensis*. These conditions are typical of Southeastern Spain.

In an arid climate system the model has a single stable equilibrium point, corresponding to the absence of biomass. In wet weather the system has again a single stable equilibrium point, corresponding now to the maximum biomass. Between these two extremes, i.e. a semi-arid climate, the system has three equilibria, two stable (one without biomass and the other with maximum biomass) and one unstable.

This model allows exploring strategies to facilitate the reforestation of semiarid areas. According to it, increase in the extinction coefficient of light and the density of leaves facilitate reforestation, while increases in the maximum evaporation rate, the mortality rate (litter formation) and the thickness of the leaves complicate reforestation. Maximum production has a more complex behavior.

1. Introduction

Desertification is the degradation of land in any drylands caused by a variety of factors, such as climate change and human activities (Geist, 2005). It is one of the most important global environmental problems, since arid ecosystems are among the most susceptible to global climate change (Schröter et al., 2005). For this reason rapid desertification is taking place in a number of regions throughout the world (Rietkerk and van de Koppel, 1997; Nicholson, 2000), as in the Mediterranean area, where there is a real risk of imminent desertification caused by an abrupt transition (Kéfi et al., 2007b).

However, this trend is not general. There is evidence that the amount of forest in Europe has been increasing in recent years (Bruinsma, 2003).

Moreover, there is evidence that degraded communities often do not respond predictably to management efforts, producing unexpected results (Hobbs and Harris, 2001). These unexpected outcomes are sometimes due to the focus on restoring the abiotic conditions of the system, and ignoring changes in biotic factors. Suding et al. (2004) examine four biotic constraints that contribute to the ecological resilience of degraded systems against restoration efforts: positive feedback between the species of the degraded state; patterns of herbivory and other trophic interactions; the lack of landscape connectivity and sufficient local seed sources; and changes in the frequency of wet/dry years or another long-term change.

A spectacular example of reforestation is the case of Sierra Espuña in the early twentieth century. Sierra Espuña is a mountain in Southern Spain, in an area with a semiarid climate. In simple terms, a semi-arid climate may be defined as one in which rainfall is between 200 and 500 mm year^{-1} (Verheye, 2009). Sierra Espuña was a semi-desert mountain in the late nineteenth century (Fig. 1), but thanks to the work of the forester Ricardo Codorniú it was reforested in the early twentieth century, and today it is an important forest where biomass is increasing spontaneously in some areas (Fig. 2). This suggests that the semi-arid climate of this area could have two types of stable equilibrium; one without vegetation, and the other with a large forest.

There are three important aspects to the modeling of the reforestation of semiarid areas: one is the interaction between different species, such that, depending on environmental conditions, it can cause competition or facilitation effects (Holzapfel et al., 2006; Pugnaire and Luque, 2001); another is the formation of spatial patterns, either by vegetation-moisture interactions that take into account overland water flow (Rietkerk et al., 2002) or by water conduction by plant roots (Gilad et al., 2004); the third is the interaction between vegetation and soil moisture with bistable behavior (Connel and Sousa, 1983; Wilson and Agnew, 1992; Rietkerk and van de Koppel, 1997; Scheffer et al., 2001; Zeng et al., 2004; Kéfi

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Fig. 1. Sierra Espuña (Spain), in the late nineteenth century, before reforestation. Courtesy of the Archives of the Dirección General de Medio Ambiente de Murcia (Spain).

et al., 2007a; D'Odorico et al., 2007). Even a climate as dry as the Sahara could be in a bistable state, such that the current climate would be consistent with an alternative state with more biomass (Brovkin et al., 1998). What all these models have in common is that they have two steady states, one without vegetation and the other with high biomass, and the transition from one to another can occur abruptly with a slight change in any of the system parameters.

The last two approaches could converge because, as pointed out recently (Bel et al., 2012), bistability of uniform and patterned states often imply the existence of a multitude of additional stable states. Moreover, these transitions may proceed gradually by local domains of one stable state expanding into the alternative stable state.

This first paper is limited to the dynamic modeling of a single species, regardless of the spatial variability; subsequent papers will deal with the other two approaches. It is based on a system with two ordinary differential equations: one which describes the dynamics of plant biomass and the second the dynamics of soil moisture. It considers only plant–water interaction because the growth of dryland plants depends fundamentally on the availability of soil moisture (Greene et al., 1994; Bhark and Small, 2003; D'Odorico and Porporato, 2006; D'Odorico et al., 2007). The model works with *Pinus halepensis*, a species that has been used for reforestation under water-stressing conditions in Eastern Spain (Maestre and Cortina, 2004), and a loamy sand soil type, which is the one with greater availability of water in this area and is quite common (García-Fayos et al., 2000). The model only works with this type of soil because the conclusions reached do not change significantly with other soil types, and because it is a first approximation.

Fig. 2. Sierra Espuña (Spain), at present, more than a century after reforestation. Photo by D. Manuel Balsalobre Ortega.

In this paper we will consider an arid climate as one in which there is no vegetation, and a wet climate as one in which vegetation is at a maximum. As the *P. halepensis* species is adapted to dry conditions, precipitation values for these types of climates are lower than those which are commonly accepted.

The phase plane of this simple model shows the following behavior: when the precipitation rate is low (arid climate) there exists a single equilibrium point without biomass. When the precipitation rate is high (wet climate) there is also a single equilibrium point with maximum biomass. Between both values of precipitation (semiarid climate) there is one unstable equilibrium point between the two stable ones. It is therefore a model with hysteresis.

This model is used to explore those features of the system that facilitate the transition from one state to another, i.e. to facilitate reforestation in a semiarid climate, considering that for reforestation to take place in this type of climate, it is only necessary to reach the point of unstable equilibrium, because from here on the system will tend to the state with maximum biomass spontaneously.

2. Materials and methods

The model consists of a system with two differential equations, the first one for the variation of plant biomass and the second one for the variation of soil moisture. Table 1 explains the meaning of each parameter of this model. Tables 2–4 sets the value of those specific parameters.

The equations are:

$$\begin{aligned} \frac{dB}{dt} &= P_{\max} GT(T) GS(S) GB(B) - \mu B \\ nZ_r \frac{dS}{dt} &= R - EB(B)ES(S) - UB(B)US(S) - L(S) \end{aligned} \quad (1)$$

where B is the biomass (g m^{-2}), S is the soil moisture (dimensionless), t is the time (day), n is the soil porosity (dimensionless), Z_r is the rooting depth (mm), $P_{\max}$ is the maximum net production per soil surface (g m^{-2} day^{-1}), $GX(X)$ is the effect of X factor on the net production (dimensionless), T is the temperature (°C), μ is the death rate (day^{-1}), R is the precipitation (mm day^{-1}), $EB(B)$ is the evaporation ratio dependent on biomass (dimensionless), $ES(S)$ is the water loss through evaporation depending on soil moisture (mm day^{-1}), $UB(B)$ is the transpiration rate dependent on biomass (dimensionless), $US(S)$ is the water loss through transpiration depending on soil moisture (mm day^{-1}), $L(S)$ is the loss of water due to leakage (mm day^{-1}).

Next we will develop each equation separately.

2.1. Soil moisture

From the system of equations in (1), we first develop the equation which measures soil moisture. This equation is inspired partly in Rodriguez-Iturbe and Porporato (2004):

$$nZ_r \frac{dS}{dt} = R - EB(B)ES(S) - UB(B)US(S) - L(S) \quad (2)$$

In this model, the parameter Z_r has a constant value of 500 mm, which is reasonable according to Rodriguez-Iturbe and Porporato (2004). However, the behavior of the model varies very little when we modify this parameter.

The first term on the right of the equation corresponds to the input of water by precipitation. The second is the output of water by evaporation. The third is the output of water through transpiration. Finally, the latter corresponds to the flow of water through runoff.