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Optimal short-term policies for protection of single biological species from local extinction

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ABSTRACT

This work introduces a finite-horizon bioeconomic growth model that links the biological evolution of a single species with the capital accumulation dynamics. The model is formulated as a problem of optimal control with non-consumptive objective regarding the biological species. The application of the Pontryagin's maximum principle allows designing a decision policy for short-term optimal planning and converts the optimal control problem to a two-point boundary value problem. The latter is then solved numerically using the MATLAB routine bvp4c. The results of numerical simulations suggest the existence of optimal policies capable to enhance even (initially) scarce species populations within a finite period of time. This supplements previous studies of various scholars where such policies were designed for infinite horizon and required initial abundance of the species.

1. Introduction

Historical evidence shows that nature evolution itself may result in extinction of some biological species (as an example, think of dinosaurs) or the latter may be caused by natural phenomena or disasters. On the other hand, almost all scientists agree that the major factors that contribute to biodiversity loss[1] are: overexploitation, deforestation, invasive species, air and water pollution, soil contamination, and climate change. It is interesting to note that, according to Ehrlich (1988), the *primary cause* of the decay of organic diversity is *not* direct human exploitation or malevolence, but the habitat destruction that inevitably results from expansion of human population and human activities.

A critical review done by Eppink and Van Den Bergh (2007) summarizes key features of four basic categories of models that integrate economic theories and strategies aimed at species conservation. The majority of these models are designed in order to help a social planner to define strategies for optimal and/or sustainable *harvesting*, where species preservation guarantees the profit stability for future generations and thus contributes to the economic development.

On the other hand, there are many wildlife species with *no harvest value* that are currently in threat by negative side-effects of human activity (urbanization, pollution, habitat loss, etc.). As pointed out by Madan and Madan (2009, p. 133) and Aggarwal (2010), most of the species that are becoming extinct are not "food species" (that is, they are not directly consumed by humans) but their biomass is converted into human food when their habitat is transformed into pasture, cropland, and orchards. A recent study carried out by Dumont (2012) indicates that we are facing a considerable reduction of the surface area of wild biodiverse land by the year 2050 as a consequence of growing human population on out planet. One may argue that wild species are not absolutely essential for human survival. However, loss of wilderness irreplaceably diminishes an important source of human wellbeing.

To study the evolution of wild endangered species, Alexander and Shields (2003) had proposed a *non-harvesting* variant of dynamic model for one particular species (New Zealand's yellow-eyed penguin) using as a control variable an index of the quantity of land resources, which are vital for the species survival. The latter can be viewed as a defensive expenditure of the society aimed at the conservation of the natural habitat of the species. In fact, this non-harvesting model does not explicitly include the negative impact that human activity and aggregated production may eventually have on the natural evolution of species population.

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[1] For more comprehensive review on imminent threats to biodiversity please refer to Madan and Madan (2009, pp. 133–135).

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However, the results of Antoci et al. (2005a,b) clearly demonstrate that both negative and positive human actions may alter the stability properties of the natural dynamic of biological species. Their studies, performed on a basis of a dynamic model of two interacting species with linear dynamics, revealed an interesting fact. Namely, if the equilibrium level of the species is high enough, the local stability's properties will be preserved when the natural biological dynamics (without human intervention) is amended with economic and ecological features (that is, negative impact of aggregated production and positive impact of defensive expenditures). Additionally, Campo-Duarte and Vasilieva (2011) and Cruz-Rivera and Vasilieva (2013) have come to the same conclusion using as a basis a single-species model with Gompertz-type and logistic population growth, respectively.

This paper intends to contribute in this strand of research by introducing and exploring a *finite-time variant* of the bioeconomic model initially proposed by Cruz-Rivera and Vasilieva (2013) for infinite time horizon. Our study is formalized by introducing a stylized bioeconomic model in finite continuous time that includes two state variables – the population of a species (biological component) and the capital (economical component). The model, formulated in Section 2, also contains two control variables – the consumption related to aggregated production with its *negative effects* (such as pollution, reduction of species habitat, etc.) and the generic defensive expenditures aimed at the conservation of a species (that is, *positive effects*). We do not consider the harvesting (i.e., direct consumption of the species) and suppose that the population dynamics of the species is altered only by *negative side-effects* of aggregated production and by *positive effects* of defensive actions aimed at prevention of single species from local extinction. Eventually, the aggregated production itself may have a circumstantial (or indirect) positive impact for the species protection. Namely, an outburst of economic activities may provide an extra capital surplus that could be further used to subsidize additional defensive actions.

We presume that there is a social planner acting in the economy who chooses the levels of consumption and investments in defensive expenditures so as to maximize the species population and capital outcome by the end of finite period of time together with overall utility subject to the physical capital accumulation dynamics and the amended ecological dynamics. The utility function considered in this paper favors consumption over the species conservation.[2]

Having defined the decision criterion, the model is then formulated in terms of optimal control (Section 3). The application of Pontryagin's maximum principle results in a 4D optimality system with two-point boundary conditions. The latter is then solved using MATLAB routine `bvp4c` that implements the *collocation method* for two-point boundary value problems (BVPs). Section 4 contains the results of numerical simulations and their interpretations.

2. Model description

Let us suppose that a single-species population $x(t)$ obeys the evolutionary biological dynamics given by logistic equation under the course of nature without human intervention. Eventually, the biological dynamics can be affected by side effects of human activity such as aggregated production, pollution, urbanization, etc. By introducing a simple growth model, the species biological evolution can be linked to a capital accumulation dynamics. For the sake of simplicity, we assume that there is a single good which is produced by capital $k(t)$ (see Antoci et al. (2005a), Campo-Duarte and Vasilieva (2011), and Cruz-Rivera and Vasilieva (2013) for more details regarding the model's description). As a result we obtain a bioeconomic ODE system

$$\begin{cases} \dfrac{dx}{dt} = rx(t)\left(1-\dfrac{x(t)}{K}\right) - \epsilon k^{\alpha}(t) + \sigma d^{\mu}(t), & x(0)=x_0, \\ \dfrac{dk}{dt} = pk^{\alpha}(t) - c(t) - d(t), & k(0)=k_0, \end{cases} \tag{1}$$

whose entries are defined in Table 1. Here $x_0 > 0$ and $k_0 > 0$ are initial values of the species population and the capital, respectively. It should be noted that parameter $\mu \in (0, 1)$ is introduced in order to emphasize that the positive effect of defensive investment on the specie evolution *is not* directly proportional to population growth; in other words, extra-spending on species conservation (increase in $d(t)$) may decrease the positivity of such impact on the evolution of $x(t)$ due to the carrying capacity limitations of the environment.

Suppose that a social planner has a short-term task to maximize the final values of the species population $x(T)$ and the capital $k(T)$, where $0 < T < \infty$ is a finite horizon measured in months. Additionally, a social planner also wants to maximize its overall utility over $[0, T]$ which can be expressed as a definite integral of a *non-consumptive* utility function $U(x, c)$ since the direct harvesting of the species is not considered in the model.

Remark 2.1. In order to avoid possible confusions, the term "non-consumptive utility" here refers principally to "non-consumptive use value" of endangered wild species in the sense of Boyle and Bishop (1987). According to Alexander (2000), the non-consumptive public good values of endangered species can be also viewed as species existence values. On the other hand, the control variable $c(t)$ refers to an instantaneous portion of capital surplus destined to multipurpose consumption of human society that does not derive any benefit from specie's harvesting.

The definition of utility function $U(x, c)$ must clearly reflect the priorities of decision-making. From the economical point of view, utility function $U(x, c)$ should be non-negative, twice differentiable, monotonically increasing and concave. Using the argument of Antoci et al. (2005a), we assume that the society derives utility from both multipurpose consumption c and species abundance x where the latter has an indirect non-consumptive value. Therefore, the non-consumptive utility of species population can be set directly proportional to x. On the other hand, the utility of consumption has logarithmic form in various economic application (see, e.g. Grossman and Helpman (1991) among other sources). Following the idea initially proposed by Antoci et al. (2005a) and further developed by Campo-Duarte and Vasilieva (2011) and Cruz-Rivera and Vasilieva (2013), we will examine following utility function:

$$U(x, c) = \eta x + \nu \ln c, \tag{2}$$

where η, $\nu > 0$ are some specified weight coefficients. Function (2) expresses that utility of consumption (logarithmic term) has decreasing utility gain for increasing consumption while the non-consumptive utility gain remains constant (linear term). It should be emphasized that ν must be significantly greater than η since linear term in (2) has higher growth rate comparing to logarithmic term. Additionally, function (2) reflects that consumption $c(t)$ is more important than species conservation. In other words, total extinction of the species ($x = 0$) can be paid off by an aggregated consumption level.

Remark 2.2. It is interesting to note that for (2) the marginal rate of substitution (MRS) is calculated as

$$\text{MRS} = \frac{U_c(x, c)}{U_x(x, c)} = \frac{\nu}{\eta c}$$

[2] The same utility function was treated by Antoci et al. (2005a), Campo-Duarte and Vasilieva (2011), and Cruz-Rivera and Vasilieva (2013) to describe a situation when biological species are threatened by local extinction.