Atmospheric Environment 128 (2016) 263–267

Contents lists available at ScienceDirect

Atmospheric Environment

journal homepage: www.elsevier.com/locate/atmosenv

Icing severity forecast algorithm under both subjective and objective parameters uncertainties

Feng Zhang [a,*], Zhen Huang [a], Huiju Yao [b], Weihao Zhai [a], Tongfeng Gao [a]

[a] Institute of Aircraft Reliability Engineering, School of Mechanics, Civil Engineering and Architecture, Northwestern Polytechnical University, Xi'an 710129, China

[b] Jiangnan Institute of Electrical and Mechanical Design, Gui Yang, 550009, China

HIGHLIGHTS

- Meteorological observation data are described as random variables.
- Monte Carlo method is performed to forecast the probability.
- Membership function is introduced to describe the subjective uncertainty.
- A probabilistic method under considering parameters uncertainties is proposed.

ARTICLE INFO

Article history:
Received 22 September 2015
Received in revised form
26 December 2015
Accepted 31 December 2015
Available online 8 January 2016

Keywords:
Icing forecast model
Icing severity level
Probabilistic forecast
Membership function
Monte Carlo simulation method

ABSTRACT

Based on the traditional deterministic methods for estimating the airplane icing severity, a probabilistic method under considering subjective and objective parameters uncertainties is proposed. In that method, all meteorological observation data are described as random variables, then Monte Carlo method is performed to forecast the probability of airplane icing. In the process of forecasting the airplane icing severity level, the membership function is introduced to describe the subjective uncertainty. Finally, the generalized probabilistic solution formula of estimating the airplane icing severity level is derived. The proposed probabilistic model and solutions for forecasting the airplane icing severity level are proved to be reasonable and applicable by a real airplane icing case.

1. Introduction

When the airplane flies in the clouds with supercooled water droplets, the supercooled water droplets will freeze in the windward surface parts rapidly and accumulate into ice, which is a serious threat to the airplane safety (Kind et al., 1998; Ratvasky et al., 2005). Mild ice can reduce the airplane's flight performance, leading to the decrease of the airplane lift and the increase of resistance, which will cause the difficulty of controlling the flight attitude. Severe icing can lead to the stall of the plane under the small angle of attack (Bragg et al., 2000). That can even cause a tragedy of fatal crash (Fuchs and Schickel, 1995). According to statistics, the probability of accident caused by the aircraft icing is more than 15%. In the recent years, aircraft icing has caused many grave accidents. For example, in June 2009, France A330 met bad weather, iced and crashed when flying over the Atlantic Ocean, killing 228 people (Li and Zhou, 2010). Therefore, the problem of airplane icing is one of the important contents in airplane design (Chen et al., 2009; Yi, 2007).

Through years of research, many methods have been developed for the airplane icing forecast. For examples, the statistical forecast equation, false frost point temperature empirical formula, the prediction method of icing intensity estimation and so on (He et al., 2012; Daniel et al., 1995; Thompson et al., 1997). Based on the parameters like liquid water content, temperature, and the median size of cloud droplets, Daniel Cornell and other people in the United States established an icing severity index to evaluate freezing strength. Considering the atmospheric temperature and the dew point temperature on flying height, the National Center for

* Corresponding author. P.O. BOX 883, Northwestern Polytechnical University(Chang'an Campus), Xi'an 710129, China.
E-mail address: nwpuwindy@mail.nwpu.edu.cn (F. Zhang).

http://dx.doi.org/10.1016/j.atmosenv.2015.12.066
1352-2310/

Atmospheric Science established Rawinsonde Observation(RAOB) icing forecast scheme. Based on the above research, domestic scholars have carried out the relevant application. Ref. (Gong, 1998) analyzed an event of a certain type of airplane during a severe icing with the statistical forecast equation and the fake frost point temperature empirical formula. Ref. (Sun et al., 2012) also developed some related airplane icing forecast applications which were meaningful to engineering practice.

It should be pointed out that, there are many parameters that can affect the airplane icing. These parameters are all treated as deterministic values in the current freezing forecast models and analysis methods. When the flight is surrounded by the supercooled water droplets in the clouds, the flight speed, and other parameters such as the height, the liquid water content and the pressure fluctuate are all within a certain range. At the same time, the sensor also has measurement errors in the measurement of these parameters (Li and Zhou, 2010). That leads to the objective uncertainty of the traditional icing prediction model. Therefore, based on the factor prediction equation and the sufficient consideration of objective uncertainty, such as the flight speed and altitude, the pressure, temperature and dew point temperature at the flight altitude, we establish the probabilistic airplane icing forecasting model and propose the corresponding computational method for freezing probability in this paper. The computational method is derived from the Monte Carlo method. In the new method, the parameters of the prediction model are treated as random variables with certainty probability density functions.

The four ice strength grades: weak, moderately strong, strong and extremely strong are given to guide the pilot for the corresponding flight operation in traditional ice strength analysis models (Yi, 2007). Due to the limitation of cognitive level, there are ambiguities among the four freezing strength grades. That is to say, there are subjective uncertainties on cognitive level in freezing strength grade assessment model (Liu, 1999, 2004). To be more close to the actual situation, in this study, we introduce the membership function to describe the subjective uncertainty of the freezing strength grade, and also give the formulas of generalized probability about freezing strength grade.

This paper is organized as follows. The freezing probability prediction model is established and the solving method is derivated in Section 2. The freezing strength grade evaluation is provided considering the subjective uncertainty in Section 3. The feasibility and rationality of the presented analysis process are verified with the examples in Section 4. Conclusions are presented in Section 5.

2. Freezing probability prediction model and solving method based on the objective parameter uncertainty

The prediction equation of airplane icing factor y is shown in Eq. (1), which can be used to assess whether the plane will freeze in bad weather conditions while flying (Gong, 1998).

$$y = 4.9335 + 0.002016V + 0.00818(P - P_0) - 4.4358f + 0.2839H \quad (1)$$

where V stands for flight speed ($km\_\ h^{-1}$), H stands for flight altitude(km), P_0 is the air pressure at $0^\circ\ C$, P is the air pressure at the height of H, and f is the relative humidity at the height of H, which can be acquired by Eq. (2).

$$f = 10^{ab(T_d - T)/(b+T_d)(b+T)} \quad (2)$$

where T_d is the dew point temperature at flying height, T is the temperature at flying height, a and b are parameters with $a = 7.5$, $b = 237.3$.

Studies have shown that $y - 2.98 \geq 0$, none-freeze; $y - 2.98 < 0$, freeze. The following Eq. (3) is the discriminant formula of airplane icing events.

$$\begin{aligned} G_1 &= y - 2.98 \\ &= 1.9535 + 0.002016V + 0.00818(P - P_0) \\ &\quad - 4.4358 \cdot 10^{ab(T_d - T)/(b+T_d)(b+T)} + 0.2839H \end{aligned} \quad (3)$$

If ($G_1 \geq 0$), none-freeze, or else, ($G_1<0$), freeze. In the traditional forecasting model, these parameters-V, P, P_0, T_d, T and H are deterministic. In actual situation, these parameters fluctuate within certain ranges during the process when the plane goes through the clouds. At the same time, the sensor also has measurement error in the measurement of these parameters. Due to these reasons, the parameters such as V, P, P_0, T_d, T and H possess random uncertainty. Therefore, the event G_1 also has objective random uncertainty. This suggests that the airplane icing phenomenon $P_{G1}(G_1 < 0) + P_{G1}(G_1 \geq 0)=1$ is in probability, which needs to be analyzed by the theory of probability event. Only in this way, we can evaluate the possibility of freezing up under the airplane flying in bad weather reasonably.

The paper uses the vector$\mathbf{x}$ to denote the parameter vector (V,P,P_0,T_d,T,H). The common type of distribution is normal distribution. The form of probability density function $h_{\mathbf{x}}(\mathbf{x})$ is shown in the following Eq. (4).

$$h_{\mathbf{x}}(x) = \frac{1}{\sqrt{2\pi}\prod_{i=1}^{6}\sigma_i} \exp\left(-\sum_{i=1}^{6}\frac{(x_i - \mu_i)^2}{2\sigma_i^2}\right) \quad (4)$$

where μ_i and σ_i are the mean and standard deviation of the *ith* component of the vector $\mathbf{x}$ respectively.

When correlation exists between random variables, the correlated variables can be transformed into independent ones through Nataf Transformation (Lu, 2007; Li et al., 2008). For non-normal random variables, they can be transformed into normal ones by Rosenblatt or Rackwitz-Fiessler Transformation (Rosenblatt, 1952; Rackwitz and Fiessler, 1978). Therefore, this article only considers independent normal variables.

By generating the samples $\mathbf{x}$ according to the probability density function $h_{\mathbf{x}}(\mathbf{x})$, we can get G_1 through Eq. (3). Further, we can get the probability value $P_{G1}(G_1<0)$ of the event ($G_1<0$) and the probability value $P_{G_1}(G_1 \geq 0)$ of the event ($G_1 \geq 0$).

Obviously, the probability values of the event ($G_1<0$) and the event ($G_1 \geq 0$) satisfy Eq. (5).

$$P_{G_1}(G_1 < 0) + P_{G_1}(G_1 \geq 0) = 1 \quad (5)$$

Freezing probability $P_{G_1}(G_1 < 0)$ is solved by integrating in the variable space, as shown in Eq. (6).

$$P_{G_1}(G_1 < 0) = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} I[\cdot] h_{\mathbf{x}}(\boldsymbol{x}) d\boldsymbol{x} \quad (6)$$

where $I[\cdot]$ is the indicator function and $I[\cdot] = \begin{cases} 0 & G_1 \geq 0 \\ 1 & G_1 < 0 \end{cases}$.

Monte Carlo method has better application in dealing with the problem of high dimension and small probability integral (Geng et al., 2013). In this paper, the Monte Carlo method is applied to estimate the probability in Eq. (6), i.e. through the joint probability density function $h_{\mathbf{x}}(\mathbf{x})$ to extract the samples, and then use them to estimate the probability $P_{G_1}(G_1 < 0)$. The estimation formula is