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Stochastic model to describe atmospheric attenuation from yearly global solar irradiation

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ABSTRACT

A new stochastic model to describe atmospheric attenuation from yearly global solar irradiation has been developed and implemented. The proposed model takes into account the consideration that the whole of all attenuating elements can be thought of as a population where the higher the number of individuals the lesser the clearness index. Thus, the inverse of the clearness index is considered as the variable of a stochastic process. From the proposed master equation as starting point, the new model is characterized by transition rates (assessed from a growing parameter - *G* - and a decreasing parameter - *D*) which depend mainly on the climatological characteristics at each location. In this sense, different regions with an attenuation level calculated from the yearly global irradiation have been established using the Köppen–Geiger climate classification as a first approach.

The model parameters *G* and *D* have been determined for different regions using the inverse of the clearness index as variable. The probability density function obtained after the application of the stochastic model for each climate zone shows how the index mode increases from the zones with lower levels of attenuation to those with higher levels of attenuation. This result confirms the proposed null hypothesis related to the use of the inverse of the clearness index as an attenuation population indicator.

The fit between the empirical data and the data provided for the model is good enough according to a Kolmogorov–Smirnov test with a significance level of 0.05. Nevertheless, it is necessary to slightly modify the climate zones of Köppen–Geiger initial classification for a better explanation of the atmospheric attenuation. This climate zones modification can be considered as an additional result.

1. Introduction

Atmospheric attenuation is due to the presence of various constituents in the atmosphere: clouds, rain, aerosols and gases. Air molecules, whose size is much lower than the wavelength of the photons, scatter the radiation (Liou, 2002). Clouds can be defined (World Meteorological Organization, 2003) as hydrometeors consisting of minute particles of liquid water, ice, or both, suspended in the free air and usually not touching the ground; they may also include larger particles of liquid water or ice as well as non-aqueous or solid particles such as those present in fumes, smoke or dust. Therefore, hydrometeors are essential atmospheric attenuators in terms of scattering and absorption (Kokhanovsky, 2003; Pyrina et al., 2015). Aerosols are particles suspended in the atmosphere (http://www.nasa.gov/centers/langley/news/factsheets/Aerosols.html; Andrews et al., 2011); when their size is sufficiently large, they do not only scatter (Liu et al., 2013), but also absorb sunlight (Meyer et al., 2013; Wang, 2013). Finally, the atmospheric gases (water vapor, ozone, carbon dioxide and oxygen) are the main absorbing gases of solar radiation (McCartney, 1983).

The whole of all these attenuating elements can be thought of as a population, consisting of very different individuals (air molecules, water vapor molecules, solid particles, rain drops, etc.). That is, each of these elements contributes to the

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attenuation, to a greater or lower extent as, certainly, the attenuation effect of each type of element is different. However, in this work, a single type of attenuating element is considered. This type does not respond to any attenuation real element, but it is an artificial unit of measure, in such a way that the higher the number of individuals in this population the higher the atmospheric attenuation.

On the other hand, atmospheric attenuation as a global process can be studied by analyzing solar irradiation at the earth's surface in relation with that at the top of the atmosphere; the latter, can easily be estimated (Iqbal, 1983; Muneer, 2004). The ratio of solar irradiation at ground to that at the top of the atmosphere is known as the clearness index (k_t). Therefore, the population of atmospheric attenuators, or number of these attenuators, should be inversely related to the clearness index, or likewise the inverse of the clearness index should be related to that population.

In this work, we have proposed that the density function of the inverse of the clearness index is characteristic for each climatic region. We have also assumed that this variable can be considered as an index related to the population of atmospheric attenuators. Thus, we have used a methodology previously applied to the distribution of populations (Marsili and Zhang, 1998), starting from a master equation that reflects the growth or decrease of these populations.

A new stochastic model, different to other stochastic parameterizations developed for atmospheric studies (Richardson, 1981; Matyasovszky and Bogardi, 1996; Kuell and Bott, 2014; Schleiss et al., 2014) is proposed in order to characterize this population, i.e., the number of attenuation elements (in terms of the inverse of the clearness index) present in the atmosphere in each climate zone.

2. A new stochastic model

The equation of radiative transfer describes the variation in the intensity of the radiation due to absorption and scattering during its passage through the atmosphere. Radiative transfer models (Clough et al., 2005; He et al., 2010; Yuan et al., 2013) divide the atmosphere in horizontal layers with a certain composition and, from the known extraterrestrial radiation at the top of the atmosphere, they solve the transfer equation in order to estimate the absorption and scattering in each layer. The general expression of the radiative transfer equation (Ren et al., 2004; Marshak and Davis, 2005) is

$$dI_\lambda = -k_\lambda \rho I_\lambda ds + j_\lambda \rho ds \tag{1}$$

where I_λ denotes the intensity of radiation (for wavelength λ); ρ is the density of the atmospheric layer whose thickness is ds. k_λ is the mass extinction cross section at wavelength λ, which is the sum of the mass absorption and scattering cross sections. Finally, j_λ is the source function coefficient.

Therefore, the variation of the intensity of radiation traversing a layer depends on two terms: a source term and an extinction term. These terms shape the changes of intensity in each layer and, thus, modify the total intensity reaching the surface.

In this work a similar procedure is proposed in order to estimate the attenuation throughout the atmosphere, but variations in time instead of variations in intensity through the layer are considered. In this sense, a stochastic model of an attenuating quantity is defined in terms of a master equation that reflects the growth or decrease of the populations (Marsili and Zhang, 1998):

$$\begin{aligned}\partial_t q_{m,t} = w_d\ (m+1)q_{m+1,t} - w_d(m)q_{m,t} \\ + w_g(m-1)q_{m-1,t} - w_g(m)q_{m,t}\end{aligned} \tag{2}$$

where $q_{m,t}$ is the average number of populations of size m at time t, and $w_g\ (m)$ and $w_d\ (m)$ are the transition rates for the growth and the decrease, respectively. Thus, $w_g\ (m)dt$ shows the probability that populations of size m increase in one member in the time interval $(t, t + dt)$, and $w_d\ (m)dt$ shows the probability that these populations lose one member during this interval.

Eq. (2) is representative of the process when $m \neq 0$; for the case $m = 0$:

$$\partial_t q_{0,t} = w_d\ (1)q_{1,t} - w_g(0)q_{0,t}. \tag{3}$$

If stationary solutions are supposed, i.e., if inputs and outputs are balanced such that $\partial_t q_{m,t} = 0$, we obtain:

$$\begin{aligned}w_d(m)q_m + w_g(m)q_m = w_d\ (m+1)q_{m+1} + w_g(m-1)q_{m-1} \\ w_d\ (1)q_1 = w_g(0)q_0\end{aligned} \tag{4}$$

where the sub-indices, referring to time, have been removed, as $q_{m,t}$ has been considered independent of time. In this work, in analogy to the formerly mentioned study (Marsili and Zhang, 1998), we have assumed the transition rates to follow a power law:

$$\begin{aligned}w_g = Gm^\alpha \\ w_d = Dm^\beta\end{aligned} \tag{5}$$

where G and D are coefficients of proportionality (related to growth and decrease) and α and β the exponents of the power relation.

The frequencies q_m corresponding to Eq. (4) are:

$$q_{m+1} = \frac{q_0}{D} w_g(0) \left(\frac{G}{D}\right)^n \frac{(m!)^\alpha}{((m+1)!)^\beta}. \tag{6}$$

Moreover, let us denote by N the total number of populations considered; in this case:

$$\sum q_m = N. \tag{7}$$

Therefore, Eq. (6) is a recursive equation (n is the number of iterations) whose parameters are, on one hand, G and α (characterizing the inputs to the system), and on the other hand, D and β (characterizing the outputs). The other parameter is the transition rate $w_g(0)$.

On the other hand, the ratio (G/D) can provide information about the relation between the inputs and outputs of attenuating elements to the system. Indeed, from Eq. (5):

$$\frac{w_g}{w_d} = \left(\frac{G}{D}\right) m^{(\alpha-\beta)}. \tag{8}$$