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A review on the applications of wavelet transform in hydrology time series analysis

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ABSTRACT

In this paper, the wavelet transform methods were briefly introduced, and present researches and applications of them in hydrology were summarized and reviewed from six aspects. They include the wavelet aided multi-temporal scale analysis of hydrologic time series, wavelet aided deterministic component identification in hydrologic time series, wavelet aided de-noising of hydrologic time series, wavelet aided complexity quantification of hydrologic time series, wavelet cross-correlation analysis of hydrologic time series, and wavelet aided hydrologic time series simulation and forecasting. Finally, several personal opinions on the possible future researches of wavelet transform and its applications in hydrology were given from three aspects: methodical researches, further applications and combination.

Contents

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1. Introduction

Hydrologic time series analysis is the central topic in stochastic hydrology, which aims at revealing complicated hydrologic processes (Rodrigueziturbe et al., 1991; Montanari and Brath, 2004; Nayak et al., 2004; Garfias-Soliz et al., 2010). It is a very important task in practice as the basis of hydrologic simulation and forecasting, water resources management, as well as many other water activities. However, hydrologic time series analysis is also a difficult task due to the influences from many unfavorable factors (Sang et al., 2009b). Enough understanding of hydrologic processes has not been gained presently, and deterministic components in observed hydrologic data are usually unknown. Moreover, noise is an inevitable part of observed data and will cause various difficulties in hydrologic series analysis, such as period identification, parameter estimation and hydrologic forecasting (Elshorbagy et al., 2002; Minville et al., 2008; Stevenson et al., 2010). Therefore, accurate results of hydrologic time series analysis cannot be easily gained (Kuczera, 1992; Schreiber, 1993; Koutsoyiannis and Montanari, 2007).

Among those methods used for time series analysis, serial-correlation analysis and Fourier transform are the most traditional and typical. When applied to hydrologic time series which operate over multi-temporal scales and show non-stationary characteristics (Padmanabhan and Rao, 1998; Cahill, 2002; Neupauer et al., 2006), these methods usually cannot meet practical needs due to their limited applicable conditions. Comparatively, wavelet analysis (WA) is a more powerful method of time series analysis. By employing WA into hydrology, it can provide an effective approach for hydrologic time series analysis (Kumar and Foufoula-Georgiou, 1993; Saco and Kumar, 2000; Gaucherel, 2002; Coulibaly and Burn, 2004; Venugopal et al., 2006; Veneziano et al., 2006). WA has been widely applied in hydrology since early studies appeared in 1990s (Foufoula-Georgiou and Kumar, 1994; Venugopal and Foufoula-Georgiou, 1996). Its better performance compared with traditional methods has been adequately demonstrated.

The basic objective of WA is to achieve a complete representation of the localized and transient phenomena occurring at different temporal scales (Meyer, 1992; Percival and Walden, 2000; Wang et al., 2007; Labat, 2008). Generally, all wavelet analyses can be divided into two types as continuous and discrete wavelet analysis. The former is to determine both the scale contents of a signal and how they vary in time. As for discrete wavelet analysis, it is generally used to decompose a series into sub-signals given proper wavelet and decomposition level, and then to guide various time series analyses, such as wavelet decomposition, wavelet de-noising, wavelet aided complexity description, and wavelet aided hydrologic forecasting (Smith et al., 1998; Whitcher et al., 2002; Partal, 2009). Several representative contributions about the applications of wavelet transform in hydrology follow. Torrence and Compo (1998) operated statistical significance testing in CWT. Labat et al. (2000) gave a detailed mathematical overview of WA. Labat (2005) reviewed the main contributions of WA in earth sciences, and Schaefli et al. (2007) summarized the applications of WA in hydrology. Sang et al. (2009a, 2010a) studied wavelet de-noising in hydrology. Coulibaly et al. (2000) used WA to select climatic patterns for regional runoff prediction. Anctil and Tape (2004) discussed discrete wavelet-based streamflow forecasting. Performances of various wavelet hydrologic models were compared in (Kisi, 2008, 2009a; Nourani et al., 2009a, 2009b).

The main objective of this paper is to summarize and review the present researches and applications of wavelet analysis in hydrology, and try to give my opinions about the possible future researches on the method. The basic concepts of wavelet transform which are widely used for hydrologic time series analysis were first described; then, present applications of wavelet analysis in hydrology were classified as six aspects, and each was summarized and reviewed. Some personal opinions on the possible further researches of wavelet analysis were presented in the final section.

2. Brief description of wavelet analysis

The continuous wavelet transform (CWT) is mainly used for revealing series' characteristics under multi-temporal scales. It is operated via the translation and dilation (or contraction) of a mother wavelet $\psi(t)$ across the series $f(t)$ as a function of time t (Percival and Walden, 2000):

$$W_f(a,b) = \int_{-\infty}^{+\infty} f(t)\psi^*_{a,b}(t)dt \text{ with } \psi_{a,b}(t) = |a|^{-1/2}\psi\left(\frac{t-b}{a}\right) \quad a,b\in R, a\neq 0 \tag{1}$$

where parameter a and b are temporal scale and time translation, respectively; $\psi^*(t)$ is the complex conjugate, and $W_f(a,b)$ is continuous wavelet coefficient, based on which the wavelet power spectrum can be computed as (Labat, 2005):

$$M(a) = \frac{1}{a}\int \left|W_f(a,b)\right|^2 db \tag{2}$$

where $M(a)$ represents the energy of series $f(t)$ under the scale a. Variation of $M(a)$ with a represents series' energy distribution under different scales. Therefore, dominant periods of series $f(t)$ can also be identified using Eq. (2), that is, the temporal scales with peak energy values are the dominant periods identified.

Observed hydrologic series in nature are usually discrete signals, thus discrete wavelet transform (DWT) is often conducted as:

$$W_f(j,k) = \int_{-\infty}^{+\infty} f(t)\psi^*_{j,k}(t)dt \text{ with } \psi_{j,k}(t) = {a_0}^{-j/2}\psi\left({a_0}^{-j}t - b_0 k\right) \tag{3}$$

in which a_0 and b_0 are constants, integer j is decomposition level, and k is time translation factor. In practice, the dyadic DWT in Eq. (4) is used commonly by assigning $a_0=2$ and $b_0=1$ (Daubechies, 1992):

$$W_f(j,k) = \int_{-\infty}^{+\infty} f(t)\psi^*_{j,k}(t)dt \text{ with } \psi_{j,k}(t) = 2^{-j/2}\psi\left(2^{-j}t - k\right). \tag{4}$$

In dyadic DWT process the first stage starts from original series, and the result includes two types of coefficient sets as "approximation" and "detail" under each level. In each stage (except the first stage) only approximation coefficients are analyzed. More details of the dyadic DWT were described in (Mallat, 1989). Wavelet and decomposition level choices are two key issues in Eq. (4). The wavelet used for DWT must