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Eulerian model of a dilute spray of charged droplets

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ABSTRACT

An Eulerian model is proposed for a dilute spray of electrically charged drops moving in a gas under the action of electric forces in conditions such that the fluctuations of the drop velocities about a local macroscopic value are small. The model consists of mass and momentum conservation equations for the drops and a Poisson equation for the electric potential of the macroscopic field induced by the charge of the drops and an externally applied voltage. When it is applicable, the model gives realistic results at a fraction of the cost of a Lagrangian simulation. Results are presented for the spray of liquid drops generated by a single electrospray source between plane parallel extractor and collector electrodes. The maximum flux of drops that can be passed from extractor to collector when the effect of the inertia of the drops is negligible is computed, and the minimum injection velocity required to prevent fly back of the drops toward the extractor for values of the flux above this maximum is determined as a function of an inertia parameter. Results for a spray with two different drop sizes show that, in agreement with experimental results published in the literature, the small drops are the first to fly back toward the extractor when the flow rate of liquid sprayed increases.

1. Introduction

Electrosprays of liquids of high electrical conductivity working in the cone-jet regime offer the unique possibility of generating nearly monodisperse sprays of micrometric and nanometric drops (Cloupeau & Prunet-Foch, 1989; Fernández de la Mora, 2007). These drops carry an electric charge, which is convenient for some applications, as the spray can be easily controlled or accelerated using electric fields. However, the charged drops also induce a field, whose effect on the dynamics of the spray and the operation of the system must be assessed. Thus, assuming that all the drops have the same electrical mobility and insignificant inertia, and that the spray originates essentially at the apex of the meniscus, which are conditions approximately realized for high conductivity liquids electrosprayed at low flow rates, Fernández de la Mora (1992) showed that the spray and the meniscus are both conical, and analytically determined the effect of the space charge associated to the drops on the angle of the meniscus (which is smaller than the angle of a Taylor cone), the distribution of spray density, and the spray current per unit solid angle. The structure of the spray in conditions where the inertia of the drops is important has been investigated by Tang & Gomez (1994). They used PDA and flash shadowgraphy techniques to measure drop sizes, axial velocities and concentrations, and proposed an Eulerian model of the spray which they used to extract the electric field from their measurements. These authors analyzed the electrostatic/inertial segregation process first noted by Zeleny (1917), whereby the primary drops carrying most of the spray mass and charge occupy a core region surrounded by a shroud of satellite drops. Gañán-Calvo et al. (1994) also used PDA techniques and gave a detailed description of the spray dynamics based on Lagrangian single-drop tracking simulations where the trajectories of

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Nomenclature

$\boldsymbol{E}$	electric field
E_0	$=V/L$
$\tilde{\boldsymbol{E}}$	$=\boldsymbol{E}/E_0$ dimensionless electric field
L	interelectrode distance
M	$=mqE_0/c_f^2L$ dimensionless inertia parameter
Q	flow rate carried by the spray
V	voltage applied between the electrodes
a	radius of a drop
c_f	$=6\pi\mu_g a$
f	distribution function
m	mass of a drop
n	number density of drops
n_0	$=\epsilon_0 E_0/qL$
$\tilde{n}$	$=n/n_0$ dimensionless number density of drops
q	electric charge of a drop
r_c	standard deviation of the injection Gaussian
$\tilde{r}_c$	$=r_c/L$ dimensionless standard deviation of the injection Gaussian
r_e	radius of the injection orifice
$\tilde{r}_e$	$=r_e/L$ dimensionless radius of the injection orifice
$\boldsymbol{v}$	macroscopic velocity of the drops
v_0	$=qE_0/c_f$
$\tilde{\boldsymbol{v}}$	$=\boldsymbol{v}/v_0$ dimensionless macroscopic velocity of the drops
v_I	injection velocity
$\tilde{v}_I$	$=v_I/v_0$ dimensionless injection velocity
$\boldsymbol{w}$	microscopic velocity of a drop
(x, r)	cylindrical coordinates (Fig. 1)
$\tilde{x}$	$=x/L$ dimensionless cylindrical coordinate
$\tilde{r}$	$=r/L$ dimensionless cylindrical coordinate

Greek symbols

ϵ_0	electric permittivity of the gas
ϕ	number of drops injected per unit time
Φ	$=\phi c_f/\epsilon_0 E_0^2 L$ dimensionless flux of drops
φ	electric potential
$\tilde{\varphi}$	$=\varphi/(E_0 L)$ dimensionless electric potential
μ_g	viscosity of the gas
ρ_g	density of the gas
ρ_l	density of the liquid

Subscripts

p	primary drop
s	secondary drop
r	ratio primary drop/secondary drop
1	reference state

individual spray drops are computed under the action of the air drag and the electric forces due to the applied field and the electrostatic interactions of all the spray drops and their image charges in the electrodes. Additional studies of various aspects of electrospray beams in a gas have been carried out by Grace & Dunn (1996), Hartmann et al. (2003), and Wilhelm et al. (2003), among others, while Gamero-Castaño (2008) used a retarding potential analyzer and an induction charge detector, along with a simplified Eulerian model, to investigate electrospray beams in vacuum.

The low values of the flow rate at which a single electrospray source can generate small and monodisperse drops has naturally led to use several sources in parallel in order to achieve a reasonable throughput, which brings the problems of charged jets and sprays interaction to the front (Almekinders & Jones, 1999; Hubacz & Marijnissen, 2003; Jaworek et al., 2006; Snarski & Dunn, 1991). Large scale compact multiplexed sources have been designed and characterized by Tang et al. (2001), Bocanegra et al. (2006), Deng & Gomez (2007), Foret & Kusy (2006), and Velázquez-García et al. (2006), for applications to particle deposition, colloid thrusters, ion beam processes, and mass spectrometry, among others. A key element of these multiplexed sources is the extractor electrode, which is usually a perforated metallic plate set at a distance from the sources of the order of their spacing and charged to a potential relative to the sources suitable to maintain stable cone-jets. The extractor serves the dual purpose of decreasing the electric interaction between sources and shielding them from the space charge of the spray that develops behind the extractor. This space charge induces an electric field directed toward the extractor, whose effect on the spray drops increases with the packing density of the sources and the electrical conductivity and flow rate of the liquid. The space-charge-induced field may cause some drops to reverse their paths and fly back toward the extractor, where they would accumulate and clog the orifices, unless it is opposed by the external field of an acceleration voltage applied between the extractor and a third collector electrode further downstream.

In this paper, an Eulerian model is proposed to numerically describe the dynamics of the spray in the space between extractor and collector. The model is intended for condition where fly back does not occur, but it may predict the onset of fly back. It is validated by comparison with experimental results and results of Lagrangian simulations carried out by Deng & Gomez (2007), and applied to analyze the spray of a single source.

2. Problem formulation

2.1. Order-of-magnitude estimations

Consider a spray of charged drops which are injected into the space between two plane parallel electrodes separated a distance L through a circular orifice of radius r_e in one of the electrodes (the extractor), as in the sketch of Fig. 1. A voltage V