Journal of Membrane Science 428 (2013) 232–240

Contents lists available at SciVerse ScienceDirect

Journal of Membrane Science

journal homepage: www.elsevier.com/locate/memsci

An analytical study of laminar concurrent flow membrane absorption through a hollow fiber gas–liquid membrane contactor

Chii-Dong Ho*, Yun-Jen Sung, Yu-Chuan Chuang

Energy and Opto-Electronic Materials Research Center, Department of Chemical and Materials Engineering, Tamkang University, Tamsui, New Taipei 251, Taiwan

ARTICLE INFO

Article history:
Received 7 September 2012
Received in revised form
24 October 2012
Accepted 26 October 2012
Available online 17 November 2012

Keywords:
Membrane gas absorption
Conjugated Graetz problem
Mass transfer
Mathematical modeling
Hollow fiber module

ABSTRACT

The performance on mass transfer for a membrane gas absorption process was investigated theoretically and experimentally in the present study. Physical absorption of carbon dioxide by water was carried out and illustrated to validate the theoretical predictions. A two-dimensional mathematical formulation was developed in a hollow fiber gas–liquid membrane contactor with gas and liquid flow rates regulated independently. The resultant partial differential equations, as referred to conjugated Graetz problems, were solved analytically using the separated variable method associated with an orthogonal expansion technique. The absorption efficiency was studied with the absorbent flow rate, gas feed flow rate and CO_2 concentration in the gas feed as parameters. Both good qualitative and quantitative agreements were found between the theoretical predictions and experimental results. The accuracy of the theoretical predictions for concurrent flow in hollow fiber gas–liquid membrane contactor is $5.18\times10^{-2}\le E\le 7.21\times10^{-2}$.

1. Introduction

Hollow fiber membrane contactors have been applied to liquid/liquid and gas/liquid applications and widely used in fermentation [1], pharmaceuticals [2], wastewater treatment [3,4], metal ion extraction [5], VOC removal from waste gas [6] and osmotic distillation [7]. The most common device of hollow fiber membrane contactors was designed in a shell-tube configuration with both shell- and lumen-side fluids flowing in parallel. Previous experimental works have reported mainly on the shell side mass transfer performance [8–10]. Some literatures were reviewed recently by Lipnizki and Field [11] and the theoretical analysis of mass transfer in the shell side was developed under the assumption of an ordered fiber arrangement [12]. Meanwhile, Chen and Hlavacek [13] and Roger and Long [14] discussed the fiber distribution and flow distribution in randomly packed fiber bundles with a fluid flowing axially between the hollow fibers.

The application of membrane contactor to gas absorption process is aiming to avoid the occurrence of foaming, unloading and flooding in the traditional dispersion module device. A hydrophobic microporous membrane material is adapted in the gas absorption process [15,16] to allow the soluble gas mixture components being selectively absorbed in the solvent on membrane surface of liquid phase. Recently, there have been many studies in liquid desiccant air dehumidification with the use of hollow fiber membrane contactors to prevent liquid droplets cross-over [17–19] in cross-flow operations [20,21]. The mass transfer resistances in liquid film and distribution coefficient of gas solute between gas and liquid phases are two important factors to estimate the overall mass transfer coefficient in physical absorption processes [22–24]. Therefore, membrane gas absorption processes have gained increasing attention in past few years [25–29] to enhance the separation efficiency of absorption systems.

The membrane gas absorption process deals with a two-phase system in which there exists a change in the composition of the gas flowing stream. Cooney et al. [30] reported a mathematical model for the mass transfer in hemodialyzers of parallel-plate and cylindrical type with constant dialysate concentration by using separation of variables and Kummer's equation. Guo et al. [31] investigated the extraction separation process of Cu^{2+} in hollow fiber membrane module with a two-dimensional mathematical model. The mathematical formulation of such coupled boundary value problem is referred to as a conjugated Graetz problem [32–34], and the analytical solution of such a conjugated Graetz problem was obtaining with the use of the orthogonal technique and separated variables method [35–38].

A mixture of carbon dioxide and nitrogen as the gas feed flows in tube side and the liquid absorbent is pure water running through the shell side. The Happel's free surface model [39], as shown in Fig. 1, was implemented to study the absorption efficiency of the shell- and tube module in hollow fiber membrane contactor in

* Corresponding author. Fax: +886 2 26209887.
E-mail address: cdho@mail.tku.edu.tw (C.-D. Ho).

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http://dx.doi.org/10.1016/j.memsci.2012.10.047

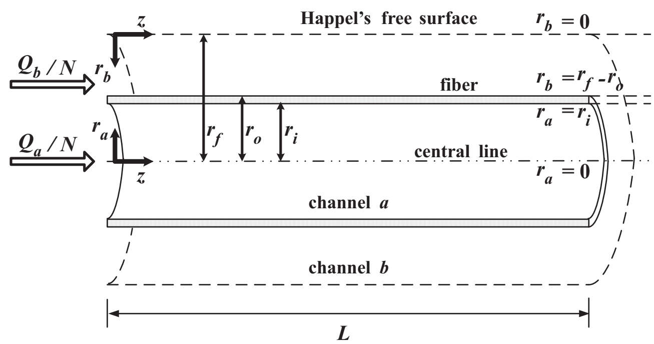

Fig. 1. Schematic diagram of Happel's free surface model in hollow fiber membrane module.

simulating the membrane gas absorption system in this work. The concentration in absorbent stream, absorption rate and absorption efficiency was investigated theoretically and experimentally with the absorbent flow rate, gas feed flow rate and CO_2 concentration in the gas feed as parameters. The theoretical predictions enable us to better understand the dominance of the various parameters in designing membrane gas–liquid contactor systems and in achieving a better device performance. It is believed that the availability of such a simplifying mathematical formulation as developed here for hollow fiber gas absorption is the value in the present work and will be an important contribution to design and analyze membrane gas–liquid contactor systems.

2. Theory

2.1. Mathematical formulations

The hollow fiber modules were divided into small cells with one fiber in each cell to conduct the shell side mass transport. The free surface appeared in the imaginary outer boundary of the cell proposed by Happel [34] and the further investigation of shell side mass transfer characteristics was developed by Zhang et al. [40]. The shell side mass transfer problem was the only issue taken into account in the cell with the assumption of the uniformed packed in hollow fiber module and ignoring the velocity profile across the module radius direction. After the following assumptions are made: steady-state fully developed laminar flow in each cell and only longitudinal component of velocity exists; negligible axial diffusion; constant physical properties in isothermal operation; the applicability of Henry's law; using Happel's surface model to characterize the velocity profile in the cell; neglecting the hollow fiber membrane thickness as compared to the hollow fiber radius, the mathematical treatments in describing the transport phenomena according to Happel's free surface model, and then, the velocity distributions in dimensionless form were obtained as

$$\left[\frac{v_a r_f^2}{LD_{AB}}\right]\frac{\partial\psi_a(\eta_a,\xi)}{\partial\xi}=\frac{1}{\eta_a}\left[\frac{\partial}{\partial\eta_a}(\eta_a\frac{\partial\psi_a(\eta_a,\xi)}{\partial\eta_a})\right] \quad (1)$$

$$\left[\frac{v_b r_f^2}{LD_{AC}}\right]\frac{\partial\psi_b(\eta_b,\xi)}{\partial\xi}=\frac{1}{(1-\eta_b)}\left[\frac{\partial}{\partial\eta_b}((1-\eta_b)\frac{\partial\psi_b(\eta_b,\xi)}{\partial\eta_b})\right] \quad (2)$$

$$v_a(\eta_a)=2\overline{v_a}\left[1-(\frac{\eta_a}{\eta_i})^2\right] \text{ for tube side} \quad (3)$$

$$v_b(\eta_b)=\frac{2\overline{v_b}}{[((2/\eta_m^2)-3)+\eta_o^2]}\left[\eta_o^2-(1-\eta_b)^2+2\ln(\frac{1-\eta_b}{\eta_o})\right] \text{ for shell side} \quad (4)$$

in which

$$\overline{v_a}=\frac{Q_a}{\pi r_f^2},\ \overline{v_b}=\frac{Q_b}{\pi r_f^2-\pi r_o^2},\ r_f=\frac{r_o}{\sqrt{\phi}},\ \eta_a=\frac{r_a}{r_f}$$

$$\eta_b=\frac{r_b}{r_f},\ \eta_i=\frac{r_i}{r_f},\ \eta_o=\frac{r_o}{r_f},\ \eta_m=\sqrt{\frac{1-\eta_o^2}{2\ln(1/\eta_o)}}$$

$$\xi=\frac{z}{L},\ \psi_a=\frac{C_a}{C_{ai}-C_{bi}},\ \psi_b=\frac{C_b}{C_{ai}-C_{bi}},\ Gz_a=\frac{\overline{v_a}r_f^2}{LD_{AB}},\ Gz_b=\frac{\overline{v_b}r_f^2}{LD_{AC}} \quad (5)$$

The boundary conditions for solving Eqs. (1) and (2) are

$$\psi_a(\eta_a,0)=\psi_{ai} \quad (6)$$

$$\psi_b(\eta_b,0)=\psi_{bi} \quad (7)$$

$$\frac{\partial\psi_a(0,\xi)}{\partial\eta_a}=0 \quad (8)$$

$$\frac{\partial\psi_b(0,\xi)}{\partial\eta_b}=0 \quad (9)$$

$$-\frac{\partial\psi_a(\eta_i,\xi)}{\partial\eta_a}=\frac{\varepsilon r_f}{\delta}\left[\psi_a(\eta_i,\xi)-\frac{1}{H}\psi_b(1-\eta_o,\xi)\right] \quad (10)$$

$$-\frac{\partial\psi_a(\eta_i,\xi)}{\partial\eta_a}=\frac{\eta_o D_{AC}}{\eta_i D_{AB}}\frac{\partial\psi_b(1-\eta_o,\xi)}{\partial\eta_b} \quad (11)$$

where ε is the permeability of membrane and H is dimensionless Henry's law constant [41]. Inspection of the resultant equations shows that boundary conditions for both channels are conjugated and the analytical solutions to this type of problem may be solved the system equations simultaneously with the use of the power-series expansion technique.

The variables are separated in the form:

$$\psi_a(\eta_a,\xi)=\sum_{m=0}^{\infty}S_{a,m}F_{a,m}(\eta_a)G_m(\xi) \quad (12)$$

$$\psi_b(\eta_b,\xi)=\sum_{m=0}^{\infty}S_{b,m}F_{b,m}(\eta_b)G_m(\xi) \quad (13)$$

where $S_{a,m}$ and $S_{b,m}$ are the expansion coefficient associated with eigenfunction $F_{a,m}$ and $F_{b,m}$, respectively, in terms of the eigenvalue λ_m, and G_m is a function of ξ and will be damped out exponentially. Substitutions of Eqs. (12) and (13) into Eqs. (1) and (2) give

$$G_m(\xi)=e^{\lambda_m\xi} \quad (14)$$

$$F''_{a,m}(\eta_a)+\frac{1}{\eta_a}F'_{a,m}(\eta_a)-\left[\frac{v_a(\eta_a)r_f^2}{LD_{AB}}\right]\lambda_m F_{a,m}(\eta_a)=0 \quad (15)$$

$$F''_{b,m}(\eta_b)-\frac{1}{(1-\eta_b)}F'_{b,m}(\eta_b)-\left[\frac{v_b(\eta_b)r_f^2}{LD_{AC}}\right]\lambda_m F_{b,m}(\eta_b)=0 \quad (16)$$

and the boundary conditions in Eqs. (8)–(11) can also be rewritten as

$$F'_{a,m}(0)=0 \quad (17)$$

$$F'_{b,m}(0)=0 \quad (18)$$

$$-S_{a,m}F'_{a,m}(\eta_i)=\frac{\varepsilon r_f}{\delta}\left[S_{a,m}F_{a,m}(\eta_i)-\frac{1}{H}S_{b,m}F_{b,m}(1-\eta_o)\right] \quad (19)$$

$$-S_{a,m}F'_{a,m}(\eta_i)=\frac{\eta_o D_{AC}}{\eta_i D_{AB}}S_{b,m}F'_{b,m}(1-\eta_o) \quad (20)$$

where the primes on $F'_{a,m}(\eta_a)$ and $F'_{b,m}(\eta_b)$ mean the differential with respect to η_a and η_b, respectively. In most cases the eigenfunction of the Graetz problem, which is transformed into an eigenvalue problem