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A new approach of direction discretization and oversampling for 3D anisotropic radiative transfer modeling

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ABSTRACT

In radiative transfer modeling, the angular variable Ω discretization can strongly influence the radiative transfer simulation, especially with small numbers of discrete directions. Most radiative transfer models use discrete ordinate method or finite volume method for solving the transport equation. Both of the methods have their own algorithms to discretize the 4π space, under the constraint of satisfying geometric symmetry and specific moments. This paper introduces a new direction discretization and oversampling scheme, IUSD, and compares it with the other methods in simulating satellite signals. This method considers the constraint of geometric shape of angular sector, and iteratively discretizes the 4π space under this constraint. The result shows that IUSD is quite competitive in the accuracy of simulating remote sensing images. Furthermore, the new method provides a flexibility for adding any oversampling angular region, with any number of additional directions, using an optimal approach in terms of the total number of directions. Several case studies are presented. It turns out that the regional oversampling has significant influence for strong anisotropic scattering. This method has been implemented in the latest code of DART 3D radiative transfer model. DART is available for scientific purpose upon request.

1. Introduction

During the recent years, the improvement of radiative transfer (RT) is usually focused on 3D landscape simulation and RT mathematical modeling. In simulating satellite signals, the angular variable Ω discretization is a much less addressed problem, although it can strongly influence the simulation of satellite signals. Therefore, the influence of the number, shape and distribution of the discrete directions on the simulation of satellite signals and 3D radiative budget is interesting to discuss. Several methods have been mentioned to study the anisotropic radiation in the field of heat and mass transfer (Modest, 2003). Among them, the discrete ordinate method (DOM) and the finite volume method (FVM) are extensively used to solve heat transfer problem in steady-state process.

DOM was firstly proposed for atmosphere radiation (Chandrasekhar, 1969) and applied in neutron transport problems (Lathrop, 1966). In recent years, DOM has been applied and optimized for heat transfer problems (Fiveland, 1984, 1987, 1988; Truelove, 1987, 1988). Its representation of 4π space is defined by a number of discrete directions that are centered on the solid angles that discretize the 4π space. The geometric shape of the solid angles is not defined. Each discrete direction is defined by its central zenith and azimuth angles (θ_c, ϕ_c), and by a weight that is specifically computed. S_n DOM approach is normally used as the discrete ordinates. The even number n can be considered as the total number of layers over the sphere ($n/2$ in upper hemisphere), where a layer is made of all angular sectors that have the same zenith coordinate. The ordinates of S_n DOM approach and their respective parameters are provided by Lathrop and Carlson (1965), which are taken as classical references. Integration over angular sectors is approximated by a numerical quadrature.

FVM (Chai et al., 1994; Chui & Raithby, 1993; Raithby & Chui, 1990) is a rather mature spatial discretization technique. Each direction is defined by exact boundaries without overlap, so a full integration over the whole sphere ensures the conservation of energy. There are several available schemes. Raithby and Chui (1990) and Chai et al. (1994) use $N_\phi \times N_\phi$ uniform angular discretization, where the zenith $(0, \pi)$ and azimuth angles $(0, 2\pi)$ are divided by N_θ and N_ϕ, respectively. This leads to a heterogeneous sampling of the 4π space with much smaller solid angles for small zenith angles. This problem is decreased by the distribution scheme called FT_n FVM (Kim & Huh, 2000): the number of discrete directions in 2 successive layers with large zenith angle is multiplied by a factor 4. Here n has the same definition as S_n DOM.

Both DOM and FVM have some strong and weak points according to application in radiative transfer. Several comparisons have been made during the recent years (Kim & Huh, 2000; Mishra et al., 2006). The

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main disadvantage of DOM arises in anisotropic situations where the distribution of discrete directions can lead to solid angles with unrealistic geometric shapes. In that case, the numerical quadrature made over a surface does not conserve the radiant energy (Raithby, 1999). Instead of using a simple quadrature, an analytical exact integration is proposed in FVM. However, the center (ordinates) and the shape of the angular sector are still not well addressed in FVM, which results in relative large error with small number of directions (Kim & Huh, 2000).

In simulating satellite signals, exact kernel and DOM are commonly used in radiative transfer models. They discretize the angular variable Ω into N directions (Ω_n, angular sector $\Delta\Omega_n$) that are the only possible directions of incident, scattered and emitted radiant fluxes (Kimes & Kirchner, 1982; Myneni et al., 1991). Here, an iterative uniform squared discretization (IUSD) method is presented. This is a new method combining the advantages of both DOM and FVM, with well defined shape and exact center of each direction. It uses the concept of "squared" angular sector with an analytical expression that allows one to construct the direction with flexible input parameters. With this approach, scattering calculation can combine both numerical quadrature and analytical integration. Different cases are investigated: optimal shape of angular sectors on the 4π sphere, oversampling of planes and angular zones, and use of directions that are not centered on their associated sectors Ω_n for more accurate RT modeling. We test it with a 3D RT model that uses discrete directions: DART (Discrete Anisotropic Radiative Transfer) (Gastellu-Etchegorry et al., 1996, 2004, 2012). This model simulates vegetation and urban radiative budget and remote sensing images of passive and Lidar systems, for any atmosphere, wavelength and experimental configuration (spatial resolution, etc.…).

2. Algorithm

2.1. Reviews of DOM and FVM

Several basic requirements must be satisfied by any direction discretization method. First, the discretization set must be completely symmetric: the distribution of directions must be invariant after any rotation of 90° along vertical axis. This requirement is necessary for heat transfer modeling along the axes of complicated grids. Moreover, it ensures the exact calculation of boundaries conditions in RT equation. Thus, the total numbers of directions by DOM and FVM are multiples of 8 (4 in upper hemisphere and 4 in lower hemisphere). Then, moments of order 0 and order 1 of both DOM and FVM must verify some equalities. Hereafter, $\hat{s}$ stands for the direction, w stands for the associated quadrature weights, and the direction indices are sorted according to zenith angle:

- Zeroth moment ensures that discretization is exact over the 4π space:

$$\int_{4\pi} d\Omega = \sum_{i=1}^{N} w_i = 4\pi. \quad (1)$$

- First moment and upper hemisphere first moment ensure the conservation of radiant energy:

$$\int_{4\pi} \hat{s}\, d\Omega = \sum_{i=i}^{N} \hat{s}_i w_i = 0 \int_{2\pi} \hat{s} d\Omega = \sum_{i}^{\frac{N}{2}} \hat{s}_i w_i = \pi. \quad (2)$$

Fig. 1 shows several 3D illustrations of the available discretizations. In the DOM, the weight w is calculated to satisfy both the zeroth and the first moments. w corresponds to a solid angle $\Delta\Omega$, but the shape of the estimated angular sector is not defined geometrically. Although the value of w can be determined, one can obtain an inadequate discretization of 4π space. For example, several solid angles may intersect each other or may not be exactly juxtaposed.

Thus, the total integration may lead to problems concerning conservation of energy. In S_n DOM approximation of the DOM (Fig. 1(a)), the sphere is quarterly divided along x and y axes. Starting with 1 direction in the first layer in upper hemisphere, each layer of the hemisphere has a number of directions which are equal to the layer index. Thus, the total number of directions is $n(n+2)$.

In the FVM method, the 4π sphere is discretized explicitly. w is directly taken as the solid angle of each direction. In order to satisfy the first moment, $\hat{s}_i w_i$ is calculated by an integration over the angular sector. Thus, the total integration ensures the conservation of energy. In $N_\theta \times N_\phi$ FVM method, the angular sectors are uniformly divided in both zenith and azimuth axes. However, the zenith and azimuth axes are not in the uniform frame. The vertex of the zenith angle is at the center of the sphere, and that of the azimuth angle is at the center of the circle of the horizontal cross section of the sphere. The zenith angle ranges from 0 to π and the azimuth angle ranges from 0 to 2π. Fig. 1(b) and (c) shows the result of $N_\theta \times N_\phi$ FVM. It can be observed that the generated angular sectors are not perfect by shape: too narrow near the pole and too wide near the equator. This problem is decreased with the FT_n FVM (Kim & Huh, 2000). Same as S_n DOM, n is the number of layers and the layer of index i contains $4i$ directions. Its result is shown in Fig. 1(d).

2.2. Definition of a square direction

A question can be raised: what is the ideal geometric shape of a discrete direction on the 4π sphere? One can think of common figures such as: a circle, an equilateral triangle, a square, or a more complicated image like a regular hexagon. By taking into account that the figure is on the sphere surface, among all the possible shapes, the square is the simplest shape for integration. Therefore, we keep "square" in our mind throughout the whole discretization process and think about how to generate a nearly square angular sector.

Let θ and ϕ be the zenith and azimuth angles in 4π space. Table 1 gives the attributes of any direction (Ω, $\Delta\Omega$). The direction center (θ_c, ϕ_c) is the first attribute to consider. A rectangle is defined by width ($\Delta\theta$) and length ($\Delta\phi$) on a spherical surface. It differs from that on a plane surface. For example, the shape of a direction near the top of the upper hemisphere is more like a triangle, or a trapezoidal rather than a rectangle. Thus, the coordinates that represent any direction (Ω, $\Delta\Omega$) correspond to the centroid of (Ω, $\Delta\Omega$), with a mean azimuth angle $\phi_c = \frac{\phi_0+\phi_1}{2}$, and with a zenith angle θ_c that differs from the mean zenith angle $\left(\frac{\theta_0+\theta_1}{2}\right)$:

$$\theta_c = \arccos((\cos\theta_0 + \cos\theta_1)/2). \quad (3)$$

With this approach, the solid angles "above" and "below" (θ_c, ϕ_c) within the solid angle (Ω, $\Delta\Omega$) are equal. In order to have a square shape, the within solid angle vertical and horizontal arc lengths that cross (θ_c, ϕ_c) must be equal. Therefore, the shape of Ω is nearly a square if the following condition is verified:

$$\Delta\theta = \sin\theta_c \Delta\phi. \quad (4)$$

2.3. Calculation of the number of zenith layers

Unlike the symmetric requirement in heat transfer, the simulation of satellite signal does not contain complex 3D grid. The complete symmetry is replaced by point symmetry relative to the center of the sphere. The directions (Ω, $\Delta\Omega$) that sample the 4π sphere are symmetric relative to the center of the sphere, in such a way that a