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A numerically efficient technique of regional gravity field modeling using Radial Basis Functions

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ABSTRACT

Radial Basis Functions (RBFs) have been extensively used in regional gravity and (quasi)geoid modeling. Reliable models require the choice of an optimal number of RBFs and of their parameters. The RBF parameters are typically optimized using a regularization algorithm. Therefore, the determination of the number of RBFs is the most challenging task in the modeling procedure. For this purpose, we design a data processing scheme to optimize the number of RBFs and their parameters simultaneously. Using this scheme, the gravimetric quasi-geoid model can be validated without requiring additional information on the quasi-geoidal geometry obtained from GPS/leveling data. Furthermore, the Levenberg–Marquardt algorithm, used for regularization, is modified to enhance its numerical performance. We demonstrate that these modifications guarantee the convergence of the solution to the global minimum while substantially decreasing the number of iterations. The proposed methodology is evaluated using synthetic gravity data and compared with existing methods for validating the RBF parameterization of the gravity field.

1. Introduction

The Earth's gravity field and (quasi)geoid can be represented by a superposition of fields produced by the Radial Basis Functions (RBFs) (Barthelmes and Dietrich, 1991). RBFs are functions of the spherical distance between two points. They have a quasi-compact support and their response decreases rapidly with the distance from their centre. Due to the characteristics of RBFs, they show flexible treatment in the regional modeling. Many studies have been done to evaluate the performance of RBF approximation of the gravity field. The point-mass kernel (Barthelmes and Dietrich, 1991; Lin et al., 2014), radial multi-poles (Foroughi and Tenzer, 2014; Marchenko, 1998; Safari et al., 2014), Poisson wavelet (Tenzer et al., 2012), and Poisson kernel (Klees et al., 2008) are examples of applicable types of RBFs in gravity field modeling. The quality of the gravity field and of (quasi)geoid models parameterized by RBFs depends on the choice of the RBF parameters and their number, and on the applied procedure for solving the problem.

The RBF parameters include centres of RBFs, RBF bandwidths (or depths), and scaling coefficients. Barthelmes and Dietrich (1991) used a non-linear optimization algorithm to fix the position of RBFs in a stable approach. They claimed that optimization of the 3D configuration of RBFs and their magnitudes at the same time minimized the number of required RBFs for modeling. Weigelt et al. (2010) and Safari et al. (2014) used the non-linear regularization algorithm of Levenberg–Marquardt to optimize the 3D position of RBFs. Weigelt et al. (2010) demonstrated that

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they avoided an over-parameterization and yielded stable observation equations by applying minimum number of base functions. Safari et al. (2014) claimed that they improved the model quality while reducing the number of RBFs significantly. Since RBF bandwidths have a more important effect on their spatial behavior, in many studies, centres and bandwidths of RBFs were determined separately by using different solver methods. Hardy and Göpfert (1975) estimated the best depth of base functions based on the number of RBFs and the extent of the study area on a sphere. Marchenko (1998) located the radial multi-poles below data points and optimized their horizontal locations using the sequential multi-pole algorithm. He determined the depth and order of each radial multi-pole whenever the covariance function of the signal in the vicinity of data point was rather matched to the shape of base functions. Klees and Wittwer (2007) and Klees et al. (2008) designed a data adaptive method to fix the centres and depths of RBFs. They located the RBFs in equiangular grids and used the generalized cross-validation technique to evaluate the depths as a function of signal variation and data distribution. Tenzer and Klees (2008) investigated the performance of the RMS minimization technique as an alternative to the generalized cross-validation technique in the optimization of the RBF depths. They found that both techniques provide very similar results; however, they showed that the generalized cross-validation technique is less efficient than the RMS minimization technique in the processing of large datasets.

The determination of the optimal number of base functions is a fundamental task in the gravity field approximation with RBFs. Selecting too many RBFs causes an over-parameterization, while a low number of RBFs cannot model the signal variations properly. The number of RBFs depends on the applied procedure for solving the unknown parameters; optimizing the 3D position of RBFs simultaneously by using a non-linear solver method can reduce their number (see Barthelmes and Dietrich, 1991; Safari et al., 2014) while using different solver methods for optimizing the centres, depths, and magnitudes requires more RBFs (see Klees et al., 2008; Tenzer and Klees, 2008). Almost in all studies related to RBF parameterization of the gravity field, the focus was given only to the optimization of unknown parameters, while the number of RBFs was determined empirically or with the use of different types of gravimetric data. Tenzer and Klees (2008) suggested that the number of RBFs should be at least 20–30% of the number of observations in flat to hilly regions. In mountainous regions, Tenzer et al. (2012) found a typical number of 70% for this ratio. However, they demonstrated that applying topographic corrections to the gravity data reduces this number to about 30%. They also claimed that after finding a suitable number of RBFs, adding more RBFs does not have a significant effect on the model's accuracy. Safari et al. (2014) used different types of gravimetric data at the same time to find the optimal number of RBFs. They claimed that in gravity field modeling with RBFs using gravity anomalies as input data, the number of RBFs can be considered as a function of the RMS of residual height anomalies. However, they did not offer an empirical methodology, but the disadvantage of this method is that it requires different types of gravimetric data for validation of the obtained results, which might not be always available.

In the inversion of gravity data to the quasi-geoid model using RBFs, a systematic bias between the geometric height anomalies (observed at GPS/leveling points) and gravimetric height anomalies (modeled by RBFs) is inevitable (Foroughi and Tenzer, 2014). Klees et al. (2008), for instance, reached a large bias of about 0.5 m. This bias might be the result of achieving a local minimum solution on gravity data and not the global one, and can be minimized by applying a reliable data processing approach. In this study, a data processing scheme is designed to justify the 3D position of RBFs and their magnitudes, while it simultaneously optimizes the number of base functions. This strategy is proposed based on the direction of the changes in parameters and spectral content of gravimetric signal. In this methodology, the Levenberg–Marquardt algorithm is chosen as the non-linear optimization method that was proposed by Marquardt (1963). The solution of this algorithm might converge to a local minimum and may not necessarily be the global minimum. However, most of the iterative regularization methods are sensitive to the initial values of unknown parameters and appropriate initial values can minimize this effect (Ortega and Rheinboldt, 1970). For this purpose, the RMS minimization technique is utilized to find the proper initial values of RBF depths, while the RBF centres are initialized in equiangular grids. It is worth mentioning that the Levenberg–Marquardt algorithm has been widely used in the gravity field modeling with RBFs (for instance, see Foroughi et al., 2013; Safari et al., 2014; Weigelt et al., 2010). In these studies, the regularization parameter was initialized with an arbitrary constant value and sequentially updated by a constant factor, which increased the number of iterations significantly. In order to improve the performance of the proposed data processing scheme, we modify the Levenberg–Marquardt algorithm by providing an appropriate formula for the initialization of the regularization parameter and suggesting a specific updating rule for this parameter. In order to evaluate the performance of the proposed methodology, synthetic gravity anomaly data are utilized for the implementations. For the localization of the gravity observations required for a regional modeling, the Remove-Compute-Restore (RCR) technique is applied to subtract the global effect of the gravity field before computations and restore it after finding the solution. Based on the numerical experiments, the following refinements are achieved due to the proposed data processing strategy and applied modifications to the optimization algorithm:

- achieving a reliable approach on choosing the optimal RBF parameters and their optimal number;
- obtaining an accurate approximation of the quasi-geoid model;
- reducing the systematic bias between geometric and gravimetric height anomalies to a few centimeters;
- obtaining all the results after several iterations.

This paper is organized in six sections. In Section 2, the RBF parameterization is described in the context of gravity