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Closed-form analytical solutions for assessing the consequences of sea-level rise on unconfined sloping island aquifers

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ABSTRACT

Closed-form analytical solutions for assessing the consequences of sea-level rise on fresh groundwater oceanic island lenses are provided for the cases of both strip and circular islands. Solutions are proposed for directly calculating the change in the thickness of the lens, the changes in volume and the changes in travel time of fresh groundwater within island aquifers. The solutions apply for homogenous aquifers recharged by surface infiltration and discharged by a down-gradient, fixed-head boundary. They also take into account the inland shift of the ocean due to land surface inundation, this shift being determined by the coastal slope of inland aquifers. The solutions are given for two simple island geometries: circular islands and strip islands. Base case examples are presented to illustrate, on one hand, the amplitude of the change of the fresh groundwater lens thickness and the volume depletion of the lens in oceanic island with sea-level rise, and on the other hand, the shortening of time required for groundwater to discharge into the ocean. These consequences can now be quantified and may help decision-makers to anticipate the effects of sea-level rise on fresh groundwater availability in oceanic island aquifers.

1. Introduction

Sea-level rise has important consequences on the fresh groundwater resources in coastal regions and in oceanic islands. These usually constitute the most populated areas worldwide; they are subjected to high levels of demand for groundwater and consequently great stress on the aquifers (Ferguson and Gleeson, 2012).

The consequences of sea-level rise on sloping coastal unconfined aquifers have been recently investigated by Ataie-Ashtiani et al. (2013) who proposed an analytical solution to calculate the change of the position of the saltwater toe with sea-level rise. Later, Chesnaux (2015) provided four closed-form analytical solutions for: (1) assessing the change in water table elevation of coastal aquifers; (2) calculating the magnitude of the change of the saltwater inland toe migration within coastal aquifers; (3) measuring the change of groundwater travel times through coastal aquifers and (4) calculating the change in the quantity (i.e., volume) of coastal fresh groundwater resources resulting from sea-level rise.

In this paper, it is proposed to conduct a similar study, however applied to the case of oceanic sloping islands. Closed-form analytical solutions are developed for the cases of both circular and strip islands, for: (1) calculating the change in thickness of the fresh groundwater lens; (2) calculating the change in volume of island fresh groundwater resources resulting from sea-level rise and (3) measuring the change of groundwater travel times through island aquifers.

Contrary to coastal aquifers where the saltwater/freshwater interface intrudes inland in the form of a saltwater toe, the model for island aquifers consists of a freshwater lens which floats on top of the denser saltwater. Consequently, in a context of sea-level rise, the change in position of the interface in an island aquifer is more aptly measured by the change in the thickness of the freshwater lens (which also induces a change in freshwater volumes and a change in groundwater travel time).

Analytical solutions are useful for practitioners, even though their application is limited by a certain number of assumptions. During the last decade, several authors have developed analytical solutions for analyzing the consequences of sea-level rise on coastal aquifers and oceanic islands. Several authors have also developed numerical modeling approaches for assessing these impacts, although these are usually site-specific. Focusing on the case of oceanic islands, several authors have proposed analytical studies for investigating the impacts of sea-level rise on sloping oceanic islands, but they did not always provide closed-from solutions that may be directly applied for calculations when the different parameters that are needed to solve them are known. Ketabchi et al. (2014) proposed an analytical solution for calculating the impacts of sea-level rise on the thickness of fresh groundwater lenses in two-layer strip and circular islands. Morgan and Werner (2014) proposed an analytical solution for calculating the change in volume of fresh groundwater lenses for strip islands only. Maas (2007) proposed an approximate formula to estimate how long it takes for a

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freshwater lens body to adjust to new conditions of seepage and recharge in a context of climate change and sea-level rise.

Based on this review, there is a need for more quantitative analysis allowing us to calculate the impact of sea-level rise, not only on the estimated extent of the groundwater lens, but also on the changes in freshwater volumes as well as changes in groundwater travel times. The solutions that are proposed in this paper are closed-form solutions, which means that they can be directly solved by using the values of the various hydrogeological parameters that are required. These solutions are new because they provide, in the case of sloping islands (and not vertical shore islands), equations that directly calculate, in a context of sea-level rise: 1 — the change in thickness of the fresh groundwater lens; 2 — the change in island groundwater travel times; and 3 — the change in coastal groundwater volumes. The impacts of climate change on groundwater travel times is a particularly important issue that has not yet been addressed, even though several authors have investigated the influence of seawater intrusion on groundwater travel times in different contexts of inland aquifers (Chesnaux et al., 2005) as well as oceanic islands and coastal aquifers (Chesnaux and Allen, 2008; Greskowiak et al., 2013).

However, some limiting assumptions must be considered for developing the different solutions; i.e., the island aquifer is unconfined and saturated, homogeneous and isotropic with a hydraulic conductivity represented by a unique value K that is constant, but that can, however, represent an equivalent average value for a system of multi-layered aquifers. Also, the aquifer is considered to be uniformly recharged and its discharge into the ocean is represented by a constant head boundary condition. In this study, the closed-form solutions are developed for a constant value of recharge; but in a context of climate change, recharge may also be subject to change.

Groundwater flow is one-dimensional and is assumed to be Dupuit-type. Also in terms of limiting assumptions, the transition zone between the freshwater lens and the saltwater is not taken into account, assuming a sharp interface between freshwater and saltwater. Indeed, the interface between freshwater and saltwater is considered to be discrete, which is a simplification compared to real situations where often, no clear interface can be detected due to the presence of a mixing zone (Ferguson and Gleeson, 2012). The analytical models in the present study did not account for groundwater mixing and diffusion, which may have important effects in certain cases. Lastly, the assumption of steady-state is useful in this study but in real situations, the salinization of the coastal aquifer is gradual and the transition zone between saltwater and freshwater can move over long periods of time.

2. Regional flow solutions

One-dimensional equations for groundwater flow in oceanic islands have been addressed by Fetter (1972) with the assumption of a deep aquifer base. The steady-state regional flow solution for oceanic islands is given in the context of Dupuit's assumptions (Dupuit, 1863): the equipotential surfaces are vertical and the velocity is uniform over the entire depth. In the case of oceanic islands, a freshwater lens floats on top of saltwater, such that the interface between freshwater and saltwater is considered as a no-flow boundary for freshwater.

2.1. Circular islands

The problem is defined using polar coordinates (r, θ) for a horizontal plane and a vertical axis of elevation at the center of the island (axis of symmetry). Circular islands have a radial symmetry, which implies that the hydraulic head does not depend on the angle θ, but only on the radial coordinate r. The up-gradient boundary is represented by the axis of symmetry (origin, $r = 0$) and consists of a no-flow boundary (vertical impermeable boundary or water divide) where the hydraulic gradient is zero. The radius of the island is noted R (Fig. 1).

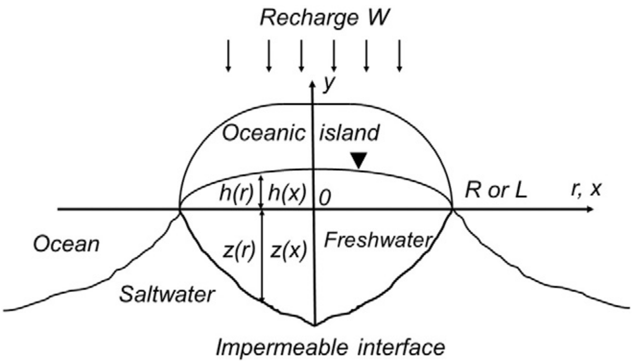

Fig. 1. Simplified conceptual model of a coastal unconfined aquifer of radius R for a circular island and of width L for a strip island.

Fig. 1 describes the oceanic island system considered. The governing equation for freshwater head is given by Fetter (1972):

$$h(r) = \sqrt{\frac{W\Delta\rho\left(R^2-r^2\right)}{2K\left(\rho_f+\Delta\rho\right)}} = \frac{\alpha}{\sqrt{2}}\sqrt{R^2-r^2} \tag{1}$$

with $\alpha = \sqrt{\frac{W\Delta\rho}{K(\rho_f+\Delta\rho)}}$ a dimensionless constantwhere ρ_f is the density of freshwater, ρ_s is the density of saltwater, and $\Delta\rho = \rho_s - \rho_f$. Note that the mean elevation of the ocean surface is taken as the datum for the heads $h(r)$. Eq. (1) was obtained by Fetter by combining Darcy's law, Eq. (2), and the Badon Ghijben–Herzberg (Drabbe and Badon Ghijben, 1889; Herzberg, 1901) relation, giving the position of the freshwater/saltwater interface. This latter relation is written as follows:

$$z(r) = \frac{\rho_f}{\rho_s-\rho_f}h(r) = \frac{\rho_f}{\Delta\rho}h(r) \tag{2}$$

where $h(r)$ is the elevation of the water table above sea level, and $z(r)$ is the depth to the fresh-saline interface below sea level.

2.2. Strip islands

The problem is defined using coordinates (x, y), but the flow is considered horizontal and one-dimensional according to the Dupuit approximation, and indicates that the hydraulic head is dependent only on x. The vertical plane, defined by $x = 0$, represents the axis of symmetry of the island and also represents the water divide (hydraulic gradient is zero), since the down-gradient boundary is a hydraulic head that is the same on both sides of the island. The length of the island is defined theoretically to be infinite; whereas its half width is L (the total width of the island is $2L$). Due to the symmetry of the island, the problem is solved between $x = 0$ to $x = L$ (Fig. 1).

Fig. 1 describes the oceanic island system considered. The solution for the elevation of the water table above sea level, $h(x)$, is given by Fetter (1972), and is expressed as follows:

$$h(x) = \sqrt{\frac{W\Delta\rho\left(L^2-x^2\right)}{K\left(\rho_f+\Delta\rho\right)}} = \alpha\sqrt{L^2-x^2}. \tag{3}$$

Note also that the means by which this equation has been obtained is similar to the method used to obtain Eq. (1) with the The Badon