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Deformation and fault parameters of the 2005 Qeshm earthquake in Iran revisited: A Bayesian simulated annealing approach applied to the inversion of space geodetic data

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ABSTRACT

The estimation of earthquake source parameters using an earth surface displacement field in an elastic half-space leads to a complex nonlinear inverse problem that classic inverse methods are unable to solve. Global optimization methods such as simulated annealing are a good replacement for such problems. Simulated annealing is analogous to thermodynamic annealing where, under certain conditions, the chaotic motions of atoms in a melt can settle to form a crystal with minimal energy. Following this, the unknown model parameters are analogous to the molecules of a molten solid whose chaotic motion gradually ceases during cooling, and the state corresponding to the global minimum of the cost function becomes highly probable at a very low temperatures.

Source parameters of the 2005 Qeshm earthquakes have already been estimated using various studies, including seismicity, the earth's surface deformation field, and rupture characteristics. Each of these studies proposes different mechanisms for the earthquakes. In this study, source parameters of the 2005 Qeshm earthquake and its main aftershock are determined with their precision by applying simulated annealing optimization in a Bayesian framework using a coseismal deformation field derived from Envisat radar interferometry. The results agree with surface ruptures and the proposed activation of the Qeshm and a NW–SE faults during main shock and main aftershock. This estimate indicates a reverse-slip of 88 ± 11 cm on the Qeshm fault and 38 ± 12 cm of strike–slip on the NW–SE fault.

1. Introduction

One of the major goals of geophysical inversion is to find earth models that explain geophysical observations. When estimating the source parameters of an earthquake, observations can include geodetic measurements of earth surface displacement obtained by GPS and InSAR. Considering the earth as an elastic half-space, Okada (1985) presented an analytical solution for surface deformation due to shear and tensile faults. Given a rectangular fault geometry (length, width, depth, strike and dip) and 3 components of dislocation (rake, slip and open), they computed displacements, tilts and strains at the free surface. Applying Okada relations which use earth surface displacement to estimate fault geometry and dislocation models, leads to a nonlinear inverse problem that requires finding the minimum of a multi-variable function. In this study, we want to minimize a cost function that characterizes the differences between observed and synthetic data calculated by the Okada relations.

Optimization schemes that use gradient information of the cost function always proceed in the downhill slope of the cost function topology (e.g., least square adjustment). Such methods are called a local optimization because they always converge to the minimum nearest to the initial model location. In case of a multimodal cost function, there is a fair chance that the convergence will lead to a local minimum unless the initial model lies within the vicinity of the global minimum. Instead, global optimization algorithms achieve convergence to the global minimum even in the presence of multimodality (e.g., Genetic algorithm [Goldberg, 1989] and simulated annealing [Ingber, 1993]). Such algorithms rely on random model perturbations, instead of information derived from the cost function, to update the model. Optimization approaches based on random perturbations of the model provide the means to jump out of local minima and potentially converge to the global minimum. These methods have been used successfully in a variety of optimization problems (e.g., Zhou and Chen, 2012; Ferreiro et al., 2012; Kim and Liou, 2013; Robini and Reissman, 2012) and have worked especially well in a variety of geophysical inverse problems

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(e.g., Faegh-Lashgary et al., 2012; Fukuda and Johnson, 2010; Koot et al., 2008).

In this study, Envisat InSAR observations are inverted to determine the source parameters of the 2005 Qeshm earthquake and its main aftershock. Different studies have been previously performed in this region based on earth surface rupture features (Shahpasand-Zadeh and Hesami, 2006), earthquake modeling using InSAR observation and seismic body waves (Nissen et al., 2007), aftershock seismicity (Gholamzadeh et al., 2007) and regional strain analysis (Amighpey et al., 2009). These studies propose different earthquake mechanisms without providing a precise criterion for the precision of the results. In this study, the inversion of an elastic half space dislocation model is performed using simulated annealing optimization combined with the Trust-Region-Reflective method (Byrd et al., 2000) in a Bayesian framework that can estimate both the source parameters of the main shock and its main aftershock as well as their estimated precision. The precision of the results is determined by computing the marginal posterior probability density of a parameter and several orders of its moments.

2. Simulated annealing method

When posed as an optimization problem, an inversion problem essentially tries to find a model that best fits the data such that the error function $E(m)$ attains a global minimum. $E(m)$ is given by:

$$E(m) = \left(-\frac{1}{2}(d_{obs} - G_m)^T C_{d_{obs}}^{-1} (d_{obs} - G_m)\right) \quad (1)$$

where d_{obs}, G, m and $C_{d_{obs}}$ are the measured data, forward modeling operator, model vector and data covariance matrix, respectively. In geophysical applications, $E(m)$ is usually a function of a large number of variables or model parameters. Local optimization always moves in the downhill direction and therefore finds the minimum closest to the starting model. Local optimization fails when the error surface has several peaks and troughs (Misra and Sacchi, 2008).

Simulated annealing (SA) is an alternative method for finding the global minimum of a function $E(m)$. In contrast to deterministic approaches, simulated annealing relies on randomly sampling the parameter space. The basic concepts of SA are borrowed from problems in statistical mechanics that involve analyzing the properties of a large number of atoms in liquid or solid samples. Physical annealing occurs when a solid in a heat bath is first heated until all of the particles are distributed randomly in a liquid phase. This process is followed by slow cooling so that all of the particles arrange themselves in a lower energy ground state where crystallization can occur. The optimization process involves simulating the evolution of the physical system as it cools and anneals into a state of minimum energy. At each temperature, the solid is allowed to reach thermal equilibrium where the probability of being in a state i with energy E_i is given by the following Gibbs or Boltzmann pdf (Landau and Lifshitz, 1980):

$$P(E_i) = \frac{\exp\left(-(E_i/KT)\right)}{\sum_{j \in S} \exp\left(-(E_j/KT)\right)} \quad (2)$$

where the set S consists of all possible configurations, K is Boltzmann's constant and T is temperature. Because SA samples models in the Gibbs' distribution, this sampler has been called a Gibbs' sampler (GS).

After reaching thermal equilibrium, the temperature is gradually reduced such that in the limit $T \to 0$, the minimum energy state becomes overwhelmingly probable. If the cooling schedule is too fast (quenching), the particles do not attain the minimum energy state and are instead trapped in a local minimum energy state forming glass. However, if the melt is cooled very slowly (annealing), then it will eventually freeze into an energy state that is at or very close to the global minimum of E, forming a crystal. The cooling schedule that is used in this study is (Ingber, 1993):

$$T_i = T_0 \exp\left(-ck^{1/NM}\right) \quad (3)$$

where T_0 is the initial temperature, k is the iteration number, NM is the dimension of parameter space and c is defined as:

$$c = m \exp\left(-\frac{n}{NM}\right) \quad (4)$$

where m and n can be considered free parameters to help tune ASA for specific applications. In this analogy, the unknown model parameters represent the molecules of a molten solid. As temperature is reduced, the chaotic motion of the molecules gradually ceases, and the state corresponding to the global minimum energy (global minimum of the cost function) becomes highly probable.

However, the Gibbs' distribution in Eq. (2) shows that to construct the pdf, we must first evaluate the partition function in the denominator of Eq. (1). This requires that the error function be evaluated at each point. However, if $E(m)$ is known at each point in the model space, then there is no need to use SA. Several computer algorithms have been proposed to avoid computing error functions at each point in the model space while still achieving an approximation of the Gibbs' pdf (Ingber, 1993). In this study, we apply a very fast simulated annealing approximation and use the Metropolis criterion to decide whether the new model should be accepted (Ingber, 1993).

In addition to finding a model that best matches the observations, it is important to describe the precision of the results. To accomplish this, we use a statistical framework to characterize the solutions by their pdfs. The statistical approach enables us to estimate uncertainty bounds on the resulting model as well as the correlation between different model parameters. Therefore, in this study, we use a Bayesian statistical framework to assess the results.

3. Bayesian framework

The Bayesian formulation was described in detail in Duijndam (1988a,b) who showed that the posterior probability density (PPD) function $\sigma(m|d_{obs})$ of model m describes the solution of the geophysical inverse problem when a Bayesian inference model is used. As the Bayes rule (Duijndam, 1988a,b), PPD is given by:

$$\sigma(m|d_{obs}) = \frac{p(d_{obs}|m)p(m)}{p(d_{obs})} \quad (5)$$

where $p(d_{obs}|m)$ is called the likelihood function denoted by $l(d_{obs}|m)$ and $p(m)$ is the probability of the model independent of the data, called the prior distribution. In geophysical inversion, the denominator term $p(d_{obs})$ is a constant. Assuming Gaussian error in observations, the likelihood function takes the following form (Duijndam, 1988a,b):

$$l(d_{obs}|m) \propto \exp(-E(m)) \quad (6)$$

Using Eqs. (5) and (6), the expression for the PPD can thus be written as:

$$\sigma(m|d_{obs}) = \frac{\exp(-E(m))p(m)}{\int dm \exp(-E(m))p(m)} \quad (7)$$

where the domain of integration covers the entire model space. In many applications, the PPD is neither analytically tractable nor easily approximated, and simple analytic expressions for the mean and variance of the PPD are not available. Even if the PPD were known, there is no way to display it in a multidimensional space. Therefore, several measures of dispersion and marginal density functions are often used to describe the answer. The marginal PPD of a particular model parameter, the posterior mean model and