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On the enhanced drag force induced by permeation through a filtration membrane

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ABSTRACT

Permeation drag is the predominant cause for particle deposition onto filtration membranes, and it is known that at close approach to the membrane surface this force may greatly exceed the Stokes drag in an unbounded fluid. Herein, the hydrodynamic interaction between a sphere and a permeable wall is re-visited within the framework of the lubrication approximation with the goal of deriving an analytical solution. A closed-form analytical solution is found, based on a perturbation expansion in terms of the scaled permeability, which is considered a small parameter. Numerical calculations of the drag force on the sphere agree perfectly with numerical results available in the literature, as do analytical model results within a range of validity which is affected by the particle size and membrane permeability. Specifically, the presented calculations have been framed in the context of the low permeabilities and colloidal particle sizes representative of commercial membrane separations in current use. Results illustrate that for reverse osmosis and nanofiltration membranes, the hydrodynamic interaction is practically identical to that of an impermeable wall, while for ultrafiltration membranes the drag is substantially reduced; an analogous trend is observed for increasing particle sizes. The approximate solution derived herein offers a simple and direct means of performing hydrodynamic force calculations in particle–membrane systems.

1. Introduction

It is well-known that a sphere approaching a solid wall interacts with it hydrodynamically, such that the drag force experienced by the sphere is greatly enhanced over that predicted by Stokes' law in an unbounded fluid. For the case of a sphere translating in the direction perpendicular to the plane wall, this problem was first fully treated by Brenner [1], who solved the equations of creeping motion using a stream-function formulation in bi-polar coordinates. Increased drag is also exerted on a sphere translating parallel to a plane wall, due to hydrodynamic interactions; however, this case is not considered herein (the interested reader is referred to, e.g., Ref. [2]).

In the considered case, that of the perpendicular approach of a sphere to the wall, the increased force is due to 'squeezing' of a liquid film between the wall and the approaching particle; at close approach, the pressure required to drive this flow becomes exceedingly large, resulting in a force on the particle which opposes its motion. The solution diverges at contact, paradoxically predicting that an infinite force would be required for the sphere to contact the surface. If, on the other hand, the wall is permeable to the surrounding fluid, this opposing force is significantly reduced since the fluid may now flow through the wall as well, as first shown by Goren [3], who also predicted that there must be a finite force at 'contact'. Goren's analysis results in a set of non-linear difference equations, which must be solved numerically; some tabulated data are provided for a range of scaled permeabilities which, at their lowest range, is comparable with a system comprised of micrometer-size particles and microfiltration membranes. Clearly, this approach is not easily accessible for consideration of membrane–particle systems representative of modern capabilities. It is also important to note that implicit to these models and, indeed, the present work, is the assumption that the wall is uniformly permeable and, hence, any local effects arising from the flow field in the vicinity of pores are ignored. This problem has been extensively treated in the literature (see, for example, Refs. [4–7]). Exact determination of this scale separation limit is complicated, since the 'pore-scale' analysis is independent of the permeability which, in turn, is the defining characteristic of a 'macro-scale' analysis for the permeable wall case. However, according to Yan et al. [4], for a particle-to-pore ratio of 10 and a separation of one particle radius, the correction coefficient to the perpendicular drag exerted on the particle deviates from that for the non-permeable case by 10%; the deviation from the 'macro-scale' permeable case is expected to be even smaller. This

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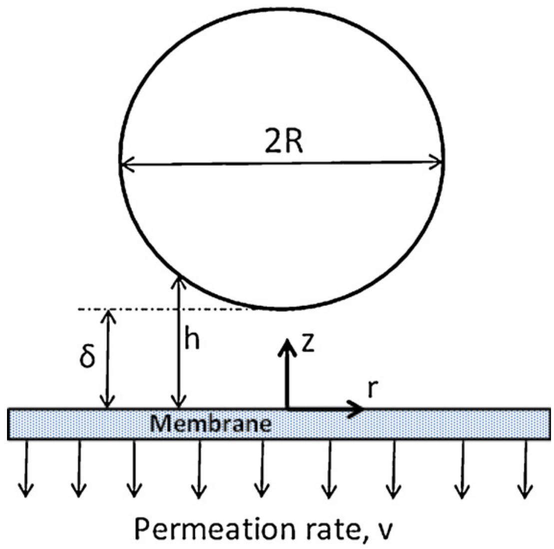

Fig. 1. Schematic illustration of the sphere-membrane geometry.

example may serve as an approximate measure of the scale separation appropriate for the type of 'macro-scale' analysis considered in this paper.

Particle deposition onto filtration membranes is primarily caused due to permeation drag, i.e., the force exerted on the particle by the flow of water permeating through the membrane. Most importantly, this enhanced drag is dependent on the membrane permeability, becoming greater as the membrane becomes less permeable. Therefore, when a force balance approach is used to model particle deposition, a drag correction factor must be used to accurately account for the permeation-induced force component. For example, Chellam and Wiesner [8] used Brenner's analytical solution to account for the increased permeation drag; this expression does not, however, account for the permeability of the membrane and so its validity for use in such a case must be established. Goren's analytical approximation for the force at contact has been used by Knutsen and Davis [9] to describe particle dynamics at the membrane surface. More recently, Goren's tabulated numerical data have been numerically approximated and employed for force balance calculations in a number of deposition studies [10–13]. This approach has been shown to produce reasonable predictions of deposition rates as well as conditions leading to reversible deposition.

While it is evident that a physically realistic drag correction for particle deposition in membrane processes must account for the membrane (wall) permeability, making such calculations requires the use of extrapolations based on tabulated data or numerically solving a set of difference equations, as formulated by Goren [3]. In this paper, the problem of a sphere interacting hydrodynamically with a permeable wall is re-visited with the goal of deriving a closed-form, approximate analytical solution, assessment of its accuracy compared with numerical calculations and previously published results and, finally, evaluation of the drag force for relevant membrane permeabilities and representative particle sizes.

2. Model formulation

Consider a spherical particle of radius R, immersed in an incompressible fluid with constant viscosity, μ, in the vicinity of a permeable membrane surface as depicted schematically in Fig. 1. The membrane is assumed to be uniformly permeable, i.e., local effects in the vicinity of pores are ignored. The fluid may be pressurized so as to induce permeation through the membrane, and this background pressure, p_∞, is taken to be everywhere uniform far from the sphere. It is further assumed that the sphere may not rotate, but may be subjected to translational motion.

The primary interest here is with the hydrodynamic interaction at close proximity of the sphere to the wall; hence, the flow within the gap between the sphere and the wall may be described using the classical 'lubrication approximation', also known as the 'long-wavelength approximation' (for a rigorous derivation the reader is referred to Refs. [14,15]). Essentially, within the lubrication framework, it is assumed that two disparate spatial scales exist, namely that the characteristic gap length-scale is much shorter than the longitudinal length-scales (radial direction, in the axisymmetric case considered here); therefore, changes over the longer length scale occur much slower than over the short scale, and so longitudinal gradients may be considered very small at leading order.

Under this approximation, the equations of motion are reduced to

$$\frac{\partial p}{\partial r} = \mu\frac{\partial^2 u}{\partial z^2}, \tag{1}$$

$$\frac{\partial p}{\partial z} = 0, \tag{2}$$

and the continuity equation

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) = -\frac{\partial w}{\partial z}, \tag{3}$$

in which u, w are the velocity components in the r and z directions, respectively, and p is the excess pressure (relative to the background pressure, p_∞).

The boundary conditions used herein are no-slip of the tangential velocity on the wall and sphere,

$$u(0) = u(h) = 0, \tag{4}$$

where $h = h(r)$ is the gap width between the sphere surface and the wall, which may be approximated as

$$h = \delta + \frac{r^2}{2R} + O(r^4), \tag{5}$$

with δ denoting the minimum distance between the sphere and the wall (see Fig. 1).

The boundary conditions on w, the z component of the velocity, are

$$w(h) = -v_s, \tag{6}$$

at the surface of the sphere, with v_s denoting the velocity of the sphere and

$$w(0) = -\frac{k^*}{\mu l}\left(p_{z=0} + p_\infty - p_p\right) = -v, \tag{7}$$

at the membrane surface, in which p_p is the pressure in the permeate side of the membrane, k^* is the Darcy permeability and l is the membrane thickness. Assuming that the permeate pressure is atmospheric, and noting that under the lubrication approximation, the pressure does not vary with z (see Eq. (2)), the velocity at the boundary may be related directly with the pressure at any radial location, viz.

$$w(0) = -\frac{k}{\mu}(p_\infty + p) = -v, \tag{8}$$

where $k = k^*/l$. Note that traditionally the permeability has the dimensions of [m^2], while in the present analysis the permeability is taken per unit thickness of the membrane, thus having the