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Obstacle avoiding patterns and cohesiveness of fish school

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HIGHLIGHTS

- We introduce a model of stochastic differential equations (SDEs) for describing the process of fish school's obstacle avoidance.
- On the basis of the model we find that there are clear four avoidance patterns, i.e., Rebound, Pullback, Pass and Reunion, and Separation, and that the emerging patterns change when parameters change.
- We present a scientific definition for fish school's cohesiveness that will be an internal property characterizing the strength of fish schooling. There are then evidences that the school cohesiveness can be measured through obstacle avoiding patterns.

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ABSTRACT

This paper is devoted to studying obstacle avoiding patterns and cohesiveness of fish school. First, we introduce a model of stochastic differential equations (SDEs) for describing the process of fish school's obstacle avoidance. Second, on the basis of the model we find obstacle avoiding patterns. Our observations show that there are clear four obstacle avoiding patterns, namely, Rebound, Pullback, Pass and Reunion, and Separation. Furthermore, the emerging patterns change when parameters change. Finally, we present a scientific definition for fish school's cohesiveness that will be an internal property characterizing the strength of fish schooling. There are then evidences that the school cohesiveness can be measured through obstacle avoiding patterns.

1. Introduction

Fish schooling, one of animal swarming, is a commonly observed phenomenon that is coherently performed by integration of interactions among constituent fish. This remarkable phenomenon has already attracted interests of researchers from diverse fields including biology, physics, mathematics, computer engineering (see Aoki, 1982; Bonabeau et al., 1999; Camazine et al., 2001; Gunji et al., 1999; Huth and Wissel, 1992; Nguyen et al., 2014; Olfati-Saber, 2006; Olfati-Saber and Murray, 2003; Reynolds, 1987; Ta et al., 2014; Uchitane et al., 2012).

Let us recall here some researches in the literature therein. In 2001, Camazine et al. (2001, Chapter 11) presented an idea on the basis of experimental result (Aoki, 1982; Huth and Wissel, 1992), and (Warburton and Lazarus, 1991) that individual fish may act following the behavioral rules:

(1) The school has no leaders and each fish follows the same behavioral rules.
(2) To decide where to move, each fish uses some form of weighted average of the position and orientation of its nearest neighbors.
(3) There is a degree of uncertainty in the individual's behavior that reflects both the imperfect information-gathering ability of a fish and the imperfect execution of the fish's actions.

Their insight is that these local rules can altogether create the coherent behavior of fish school.

Vicsek et al. (1995) modeled the movement as self-driven particles obeying some difference equations. They assumed that each individual is driven with a sum of an absolute velocity and

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an averaged velocity of nearby particles together with some random perturbations. Oboshi et al. (2002) also modeled schooling by some difference equations setting a rule that each fish choose one way of action among four possibilities according to a distance to the closest mate. Meanwhile, Olfati-Saber (2006) and D'Orsogna et al. (2006) independently presented differential equation models, but deterministic ones, utilizing the generalized Morse function and attractive/repulsive potential functions, respectively. Gunji et al. (1999) considered dual interaction which produces territorial and schooling behavior. Reynolds (1987) introduced some simple behavioral rules of animals, which are similar to (a), (b), (c) but deterministic ones, and are schematic rather than physical.

A stochastic differential equation (SDE) model describing the process of schooling was presented in Uchitane et al. (2012), where we used the above-mentioned behavioral rules (a), (b), and (c). We then utilized the model for developing quantitative arguments on fish schooling in Nguyen et al. (2014).

In the real world, the environment surrounding fish school often includes other components such as obstacles, food resources, predators, etc. In those situations, fish exhibit more complex, parallel movements such as obstacle avoidance, food finding, escaping from predator. It is evident that when a school of fish is tackled by obstacles or is hunted by predators, fish individually react quickly for avoiding obstacles or predators.

Olfati-Saber and Murray (2003) developed the method of Reynolds (1987) by introducing a dynamic graph of agents in the presence of multiple obstacles. The agents are split into several groups while approaching the obstacles. After passing all obstacles, they rejoin into a single group. Chang et al. (2003) introduced techniques of using gyroscopic forces for multi-agent systems by which the agents perform collision avoidance toward obstacles. A similar result has been shown (i.e., agents are separated into some clusters and then rejoin into a single flock). In the meantime, Hettiarachchi and Spears (2005) used virtual physical forces in composing a swarm system of robots moving toward a goal through obstacle fields. Robots may collide with obstacles but then they can still move toward the goal.

In the meantime, a concept concerning animal swarming or grouping, namely cohesiveness has already been introduced since 1930s. The study during long years seems to show that it is not an easy problem to define a concept of the cohesiveness precisely and consistently. It has been conceptualized in various ways, but each was based on intuitive assumptions and interpretations.

For instance, Moreno and Jennings (1937) defined cohesiveness as the forces holding the individuals within the group to which they belong. French (1941) noted that the group exists as a balance between cohesion and disruptive forces. Not until 1950 was a systematic theory of group cohesiveness constructed by Festinger et al. (1950). Their definition of cohesiveness is "We shall call the total field of forces which act on members to remain in the group the "cohesiveness" of that group". Gross and Martin (1952) claimed that this definition is inadequate, and they proposed an alternative definition as the resistance of group to disruptive forces. Contemporary works almost characterize group cohesion in the same way (see Hogg, 1992). Carron (1980) defined cohesiveness to be the adhesive property of group. Schachter et al. (1951) found that interpersonal attraction is the cement binding group members together. For the general relationship between cohesiveness and group performance, we refer the reader to Beal et al. (2003), Laurel (1988), and Mullen and Copper (1994).

We can however find a point of view which is common in those definitions. That is the bond linking group members to others and to the group as a whole. We believe that this common point of view may be a key feature for all intercommunicated multi-agent systems.

The objective of the present paper is two-fold: namely, studying the fish schooling from a viewpoint of pattern formation of biological systems and introducing a scientific definition of the school cohesiveness.

For the first objective, obstacle avoiding patterns of fish school are studied by newly introducing a behavioral rule for avoidance and adding its effect to our model in Uchitane et al. (2012). It is then observed that there are at least four obstacle avoiding patterns of school, i.e., Rebound, Pullback, Pass and Reunion, and Separation which are performed just by tuning modeling parameters.

For the second objective, we consider school cohesiveness as its ability to form and maintain the school structure against the white noises affecting the school. It is therefore defined as an internal nature of the school, independent of external effects. We then show how internal parameters contribute to the school's cohesiveness. Furthermore, our results suggest a very interesting correlation between the degree of cohesiveness and the four avoidance patterns.

The outline of this paper is as follows. Section 2 gives model description. We first recall the SDE model for fish schooling introduced in Uchitane et al. (2012), then newly inoculate a mechanism for obstacle avoiding into it. Section 3 presents four obstacle avoiding patterns. We thereafter investigate how these patterns change as the modeling parameters are tuned. Section 4 explores fish school cohesiveness. A scientific definition and measurement of cohesiveness are introduced. The relationship between avoidance patterns and school cohesiveness is then investigated. The paper concludes with some discussions of Section 5.

2. Model description

In Uchitane et al. (2012), we introduced a SDE model of the form

$$\begin{cases} d\mathbf{x}_i(t) = \mathbf{v}_i dt + \sigma_i dw_i(t), \\ d\mathbf{v}_i(t) = \Bigg\{ -\alpha \displaystyle\sum_{j=1, j\neq i}^{N} \left(\frac{r^p}{\| \mathbf{x}_i - \mathbf{x}_j \|^p} - \frac{r^q}{\| \mathbf{x}_i - \mathbf{x}_j \|^q} \right)(\mathbf{x}_i - \mathbf{x}_j) \\ \qquad -\beta \displaystyle\sum_{j=1, j\neq i}^{N} \left(\frac{r^p}{\| \mathbf{x}_i - \mathbf{x}_j \|^p} + \frac{r^q}{\| \mathbf{x}_i - \mathbf{x}_j \|^q} \right)(\mathbf{v}_i - \mathbf{v}_j) \\ \qquad + F_i(\mathbf{x}_i, \mathbf{v}_i) \Bigg\} dt, \quad i = 1, 2, \ldots, N, \end{cases} \tag{1}$$

for an N-fish system moving in the space $\mathbb{R}^d$ (d=2,3). Here, $\mathbf{x}_i(t)$ and $\mathbf{v}_i(t)$ denote position and velocity of the i-th fish at time t, respectively. And $\|\cdot\|$ denote the Euclidean norm of a vector, hence $\| \mathbf{x}_i - \mathbf{x}_j \|$ represents the distance between the i-th and the j-th fish.

We regarded each fish as a moving particle in $\mathbb{R}^d$. The first equation is a stochastic equation for the unknown $\mathbf{x}_i(t)$, where $\sigma_i dw_i$ denotes noise resulting from the imperfectness of information-gathering and action of the i-th fish. In fact, $w_i(\cdot)(i = 1, 2, \ldots, N)$ are independent d-dimensional Brownian motions on some probability space.

The second equation is a deterministic equation for $\mathbf{v}_i(t)$, where $1 < p < q < \infty$ are fixed exponents; α and β are positive coefficients of attraction and velocity matching among fish, respectively. And $r > 0$ is a fixed number. If $\| \mathbf{x}_i - \mathbf{x}_j \| > r$ then the i-th fish moves toward the j-th. To the contrary, if $\| \mathbf{x}_i - \mathbf{x}_j \| < r$ then the i-th fish acts in order to avoid collision with the j-th, r being thereby a critical distance (for details, see Uchitane et al., 2012).

Velocity matching of the i-th fish to the j-th also has a similar