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Toward an optimal design principle in symmetric and asymmetric tree flow networks

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HIGHLIGHTS

- Fluid flow in symmetric and asymmetric tree-shaped flow networks is studied.
- Flows of Newtonian and non-Newtonian are studied.
- Scaling laws for optimal sizes of symmetric bifurcations are proposed.
- Scaling laws for optimal sizes of asymmetric bifurcations are proposed.
- Hess–Murray's law is justified and extended.

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ABSTRACT

Fluid flow in tree-shaped networks plays an important role in both natural and engineered systems. This paper focuses on laminar flows of Newtonian and non-Newtonian power law fluids in symmetric and asymmetric bifurcating trees. Based on the constructal law, we predict the tree-shaped architecture that provides greater access to the flow subjected to the total network volume constraint. The relationships between the sizes of parent and daughter tubes are presented both for symmetric and asymmetric branching tubes. We also approach the wall-shear stresses and the flow resistance in terms of first tube size, degree of asymmetry between daughter branches, and rheological behavior of the fluid. The influence of tubes obstructing the fluid flow is also accounted for. The predictions obtained by our theory-driven approach find clear support in the findings of previous experimental studies.

1. Introduction

The phenomenon of generation of flow configuration is ubiquitously in animate and inanimate systems, both in small and large scale systems (Pries et al, 1995; Bejan, 2000; Losa et al., 2002; Lorente and Bejan and, 2008; Lorthois and Cassot, 2010; Flores et al. 2013; Razavi et al., 2014; Lorenzini et al., 2014; Miguel, 2006, 2013a; Schwen et al. 2015). Tree-shaped flow networks are commonplace in fluid dynamics systems and a topic of much current interest in many fields of science and technology (Pries et al, 1995; Bejan, 2000; Losa et al., 2002; Bejan and Lorente, 2008; Miguel, 2013b).

These networks are used for distribution and collection of fluid with the aim of providing an easier access to the currents that flow through it. Therefore, they should conform to a number of design principles to operate efficiently (Pries et al, 1995; Bejan, 2000).

A dichotomous branching of tubes typifies these networks: a parent tube divides into two daughter tubes, and the branching defines the beginning of a new generation. Another distinctive feature of these networks is their hierarchical geometry, i.e., as the tree network progresses, the vessels become narrower. Hess (1914) and Murray (1926) arrived to the conclusion that the diameters of the parent-daughter tubes obey a third-power rule. The relationship referred to as Hess–Murray's law, states that the cube of the diameter of the parent tube equals the sum of the cubes of the diameters of the daughter tubes. This relationship was first obtained based on minimization of a cost function expressed by the sum of dissipative work done by viscous forces in the fluid flow and the energy required to maintain the vascular volume (Murray, 1926).

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The constructal law is about the origin of the generation of geometric form in flow systems (Bejan, 2000; Bejan and Lorente, 2008, 2013; Miguel, 2006, 2013a). According to this law, shape does not develop by chance and a system such as area/volume-to-point flows (or vice-versa) evolves in such a way that its architecture is the one that provides easier flow access under the given constraints. Bejan et al. (2000) relied on the constructal law to derive a relationship between the length of the parent tube and the optimal lengths of the daughter tubes in a branching network. They concluded that the lengths of the daughter tubes should also obey to a third-power rule similar to Hess–Murray law. This result is obtained based on the assumption that the flow through the branching network is described by the Hagen–Poiseuille law.

Many other basic features of tree network design are explained and put on a unifying theoretical basis provided by the constructal law. Work carried out by Bejan and co-authors, derived the relationships that exist between the sizes of parent and daughter tubes under turbulent flow and also highlighted the importance of scaling principles in the design of natural and artificial vasculature (Lorente and Bejan, 2009; Zhang et al., 2009; Lee et al., 2009; Cetkin et al., 2012; Razavi et al., 2014).

Shear stress from fluid flow has a fundamental role in long-term maintenance of the structure and function of blood vessels (Rodbard, 1975; Lu and Kassab, 2011). A direct consequence of the Hess–Murray law is that wall shear stress should be the same for parent and daughter tubes. However, not uniform shear stresses have been observed in throughout the vascular system (Kamiya et al., 1984; Pries et al., 1995). Blood vessels react chronically to the mechanical forces exerted by the flowing blood (Pries et al., 1995, 2003). Long-term effects of increased blood flow on peripheral resistance, and structural adaptation of microvascular networks in response to changes in blood flow are important issues to be analyzed. An extensive experimental study on geometrical and mechanical data in the mesenteric vasculature of anesthetized rats was conducted by Pries et al. (1995, 2001). A model that is able to reproduce experimentally observed structural changes in the mesenteric circulation of hypertensive rats was also presented (Pries et al., 2005).

The vast majority of studies examining tree flow networks use Newtonian fluids. However, there is the recognition that body fluids (e.g., blood) and fluids used in industrial applications often exhibit non-Newtonian (non-linear) behavior (Schreiner et al, 1997; Chhabra and Richardson, 1999; Baieth, 2008; Dong et al., 2013). Non-Newtonian rheology has a significant impact on flows in narrow-diameter tubes. A class of non-Newtonian fluids can be defined via the following rheological law, $\tau = M\gamma^{\omega}$, which relates the shear stress in the fluid τ to a certain power of the shear strain rate γ. The power-law exponent ω is called fluid behavior index and M is the consistency index. When $\omega = 1$, the equation becomes the constitutive equation of Newtonian fluid. For $\omega < 1$ the fluid exhibits shear-thinning properties. For a shear-thickening fluid, the index ω will be greater than unity.

Revellin et al. (2009) presented an extension of Murray's law for a non-Newtonian blood flow model assuming pumping power, volume and surface constraints. They also concluded that entropy generation between the parent and daughter vessels is smaller for a non-Newtonian fluid than for a Newtonian fluid.

Most branching flow studies available in the literature assume a symmetric or quasi-symmetric branching (Schreiner et al., 1996; Schreiner et al., 1997; Kaimovitz et al, 2008; Bejan and Lorente, 2008; Miguel, 2013b, 2015; Kasimova et al., 2014). Asymmetry means daughter branches with different sizes. Binary flow trees in which the daughter tubes, at the same level of branching, have different sizes, have been observed in various natural flow systems across the biological and non-biological systems (Schreiner et al., 1996; Kaimovitz et al., 2008).

Tree flow networks of small tubes, such as that of the capillary network of blood vessels and the bronchial respiratory tree, may present significantly asymmetrical branching patterns (Horsfield et al., 1971; Phillips and Kaye, 1995; Schreiner et al., 1997; Sakaguchi, 2014). Asymmetry means also unequal distribution of fluid flow within the daughter tubes, which consequently affects the transport processes within them. So, the knowledge of how the asymmetric branching affects the flow resistance locally and, in the entire network is very important.

In this paper, we address the problem of forcing a power-law fluid through binary symmetric and asymmetric structured tree networks. The hydrodynamic performance of these structures is studied and the relationship between the sizes of the tubes in the bifurcation is examined. The effect of the degree of asymmetry over each bifurcation and on overall performance of the tree network is also investigated.

2. Theory

2.1. Best size of a system composed by a tube that feeds into two daughter tubes for easier flow access

Tree-shaped flow networks are complex branched distribution system. This branching is essentially dichotomous (Fig. 1) and is repeated for a number of generations. According to Eq. (A4) in the Appendix, the flow resistance, R_h, of an individual tube is

$$R_{\mathrm{h}} = \frac{\Delta p}{Q^{\omega}} = \frac{4M\left[8\left(\frac{1}{\omega}+3\right)\right]^{\omega} L}{\pi^{\omega} D^{3\omega+1}} \tag{1}$$

where Q is the volumetric flow rate, ω is the fluid behavior index, M is the fluid consistency index, and D and L are the diameter and the length of the tube, respectively.

For a binary tree, pursuing the analogy with electricity, the total flow resistance of the branching tubes (Fig. 2) is

$$R_{h,\mathrm{tot}} = R_{h,\mathrm{pr}} + \left(\frac{1}{R_{h,\mathrm{dga}}} + \frac{1}{R_{h,\mathrm{dgb}}}\right)^{-1} = \frac{4M\left[8\left(\frac{1}{\omega}+3\right)\right]^{\omega}}{\pi^{\omega}} \frac{L_{\mathrm{pr}}}{D_{\mathrm{pr}}^{3\omega+1}} + \frac{4M\left[8\left(\frac{1}{\omega}+3\right)\right]^{\omega}}{\pi^{\omega}} \frac{\frac{L_{\mathrm{dga}}}{D_{\mathrm{dga}}^{3\omega+1}} \frac{L_{\mathrm{dgb}}}{D_{\mathrm{dgb}}^{3\omega+1}}}{\frac{L_{\mathrm{dga}}}{D_{\mathrm{dga}}^{3\omega+1}} + \frac{L_{\mathrm{dgb}}}{D_{\mathrm{dgb}}^{3\omega+1}}} \tag{2}$$

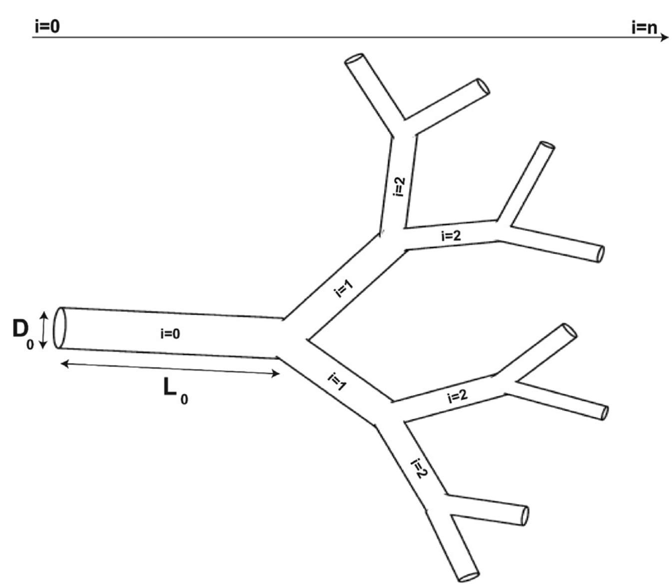

Fig. 1. Dichotomous tree flow network.