Journal of Theoretical Biology 386 (2015) 55–61

Contents lists available at ScienceDirect

Journal of Theoretical Biology

journal homepage: www.elsevier.com/locate/yjtbi

A study of autorotating plant seeds

S. Sunada [a,*], A. Ide [a], Y. Hoshino [a], M. Okamoto [b]

[a] Osaka Prefecture University, Japan
[b] Kanazawa Institute of Technology, Japan

HIGHLIGHTS

- We analyzed the flight date of rotary seeds obtained by Azuma and Yasuda (1989).
- We made flight tests of model rotors, which have various aspect ratios, airfoil shapes and total masses.
- We revealed that the lift–drag ratio affects the descent factor more strongly than the vertical force coefficient.

ARTICLE INFO

Article history:
Received 19 April 2015
Received in revised form
25 August 2015
Accepted 27 August 2015
Available online 14 September 2015

Keywords:
Leading edge vortex
Descent factor
Lift–drag ratio
Vertical force coefficient

ABSTRACT

A leading edge vortex exists on the upper surface of an autorotating plant seed. The vortex enhances the vertical aerodynamic force acting on the seed and decreases the rate of descent. We analyzed the flight data of rotary seeds and revealed that the lift–drag ratio affects the descent factor more strongly than the vertical force coefficient. This has also been confirmed by falling tests of model rotors, which have various aspect ratios, airfoil shapes and total masses.

1. Introduction

A leading edge vortex exists on the upper surface of rotary seeds (Lentink et al., 2009). The vertical aerodynamic force is enhanced by the leading edge, which decreases the pressure of the surface on the leading edge vortex. Thus, just the vertical force coefficient is focused on for decreasing the rate of descent of the rotary seeds. However, the lift–drag ratio possibly affects the rate of descent. This is because a large drag decreases the rotational speed of the rotary seeds. In this paper, the effect of the lift–drag ratio on the rate of descent of rotary seeds and model rotors is investigated.

2. Theory

Referring to Fig. 1, an angle of attack at $r = kR (0 \le k \le 1)$ is

$$\alpha = \theta + \theta_1, \theta_1 = \tan^{-1}\{(V_d - v_i)/r\Omega\}. \quad (1)$$

Here, v_i is induced velocity, which is constant at any r. It is given by Azuma and Yasuda (1989):

$$W = mg = 2\rho\pi R^2 (V_d - v_i) v_i. \quad (2)$$

This equation has following two solutions.

$$v_i/v_h = \frac{V_d/v_h \pm \sqrt{(V_d/v_h)^2 - 4}}{2} \quad (3)$$

Here, v_h is an induced velocity at hover. As stated later, the induced velocity in the present analysis is a smaller solution of this equation. This equation indicates

$$V_d \ge 2v_h \ge 2v_i, \quad (4)$$

which will be shown in Fig. 4. Equations about a vertical force and a moment around a rotational axis acting on rotary wings are given by

$$mg = \frac{n}{2}\rho \int_{R_0}^{R} c\sqrt{(r\Omega)^2 + (V_d - v_i)^2}\{(r\Omega)C_L + (V_d - v_i)C_D\}dr, \quad (5)$$

$$0 = \rho \int_{R_0}^{R} c\left\{(r\Omega)^2 + (V_d - v_i)^2\right\} r (C_L \sin\theta_1 - C_D \cos\theta_1) dr. \quad (6)$$

* Correspondence to: 1-1, Gakuanmachi, Nakaku, Sakai, Osaka, 599-8531, Japan..
E-mail address: sunada@aero.osakafu-u.ac.jp (S. Sunada).

http://dx.doi.org/10.1016/j.jtbi.2015.08.027
0022-5193/

Nomenclature

AR	aspect ratio
A_{11}, A_{12}, A_{22}	coefficients in Eq. (7)
c	chord length [m]
C_D	drag coefficient
C_L	lift coefficient
C_N	normal force coefficient
C_V	vertical force coefficient
$C_{V,ave}$	averaged vertical force coefficient
DF	descent factor
g	acceleration of gravity [m/s^2]
H	length of carbon bar [m]
k_g	$=(V_d-v_i)/\Omega R_g$
m	mass [g]
n	number of blades
r	radial position [m]
R	radius [m]
R_0	offset [m]
Re	Reynolds number
R_g	radius of rotation [m]
S_W	wing area [m^2]
U	wind velocity shown in Fig. A-1 [m/s]
v_h	induced velocity at hover [m/s]
v_i	induced velocity [m/s]
Vc	rate of climb [m/s]
V_d	rate of descent [m/s]
W	weight [kg m/s^2]
α	angle of attack [deg]
β	$=(V_d-v_i)/R\Omega$
θ	pitch angle [deg]
θ_0	angle shown in Fig. 6[deg]
θ_1	$=\tan^{-1}\{(V_d-v_i)/r\Omega\}$ [deg]
ν	kinematic viscosity [m^2/s]
ρ	atmospheric density [kg/m^3]
σ	solidity, $=nc/\pi R$
Ω	rotational velocity [rad/s]
*	values when induced velocity is ignored

When c, C_L and C_D are their values averaged along the r axis, the following equations are obtained from these equations,

$$C_L=\frac{A_{22}}{\beta A_{11}A_{12}+A_{11}A_{22}}\frac{mg}{n\rho c\Omega^2R^3},\quad C_D=\frac{\beta}{\beta A_{12}+A_{22}}\frac{mg}{n\rho c\Omega^2R^3}. \tag{7}$$

Here,

$$A_{11}=\frac{1}{3}\left[\left(1+\beta^2\right)^{3/2}-\left\{(R_0/R)^2+\beta^2\right\}^{3/2}\right]\approx\frac{1}{3}\left\{\left(1+\beta^2\right)^{3/2}-\beta^3\right\},$$

$$A_{12}=\frac{\beta}{2}\left[\sqrt{1+\beta^2}-(R_0/R)\sqrt{(R_0/R)^2+\beta^2}-\beta^2\right.$$
$$\left.\log\frac{(R_0/R)+\sqrt{(R_0/R)^2+\beta^2}}{1+\sqrt{1+\beta^2}}\right]\approx\frac{\beta}{2}\left\{\sqrt{1+\beta^2}-\beta^2\log\frac{\beta}{1+\sqrt{1+\beta^2}}\right\},$$

$$A_{22}=\frac{1}{8}\left[\sqrt{1+\beta^2}\left(2+\beta^2\right)-(R_0/R)\sqrt{(R_0/R)^2+\beta^2}\left\{2(R_0/R)^2+\beta^2\right\}\right.$$
$$\left.+\beta^4\log\frac{(R_0/R)+\sqrt{(R_0/R)^2+\beta^2}}{1+\sqrt{1+\beta^2}}\right]$$
$$\approx\frac{1}{8}\left\{\sqrt{1+\beta^2}\left(2+\beta^2\right)+\beta^4\log\frac{\beta}{1+\sqrt{1+\beta^2}}\right\},\quad\beta=(V_d-v_i)/R\Omega.$$

Then, lift–drag ratio is

$$C_L/C_D=\frac{3\left[\left(2+\beta^2\right)\sqrt{1+\beta^2}-(R_0/R)\left\{2(R_0/R)^2+\beta^2\right\}\sqrt{(R_0/R)^2+\beta^2}+\beta^4\log\frac{(R_0/R)+\sqrt{(R_0/R)^2+\beta^2}}{1+\sqrt{1+\beta^2}}\right]}{8\beta\left[\left(1+\beta^2\right)^{3/2}-\left\{(R_0/R)^2+\beta^2\right\}^{3/2}\right]}$$
$$\approx\frac{3\left[\left(2+\beta^2\right)\sqrt{1+\beta^2}+\beta^4\log\frac{\beta}{1+\sqrt{1+\beta^2}}\right]}{8\beta\left[\left(1+\beta^2\right)^{3/2}-\beta^3\right]}\equiv F(\beta). \tag{8}$$

Fig. 2 shows the relation between β and C_L/C_D. As β increases, the C_L/C_D decreases.

In a steady vertical descent, the vertical component of an aerodynamic force acting on a seed is equal to its gravitational force. Then, a vertical force coefficient $C_{V,ave}$ is given by

$$C_{V,ave}=\frac{mg}{n\int_{R_0}^{R}1/2\rho\left\{(V_d-v_i)^2+\Omega^2r^2\right\}c(r)dr}$$
$$=\frac{mg}{1/2\rho\left\{(V_d-v_i)^2+\Omega^2{R_g}^2\right\}S_W}$$
$$=\frac{W}{1/2\rho\left(1+1/{k_g}^2\right)(V_d-v_i)^2S_W}. \tag{9}$$

Here,

$$k_g=(V_d-v_i)/\Omega R_g,\quad {R_g}^2=\frac{n\int_{R_0}^{R}r^2cdr}{S_W},\quad S_W=n\int_{R_0}^{R}cdr.$$

Eqs. (7) and (8) were obtained when c is its averaged value along the r axis. However, a common value of c cannot satisfy both equations of R_g and S_W. Then, some of DF, k_g and $C_{V,ave}$, which will be obtained under a constant c, will have some errors for rotary seeds.

From Eqs. (2) and (9),

$$v_i=\frac{1}{2\sqrt{2}}\left(\frac{S_W}{\pi R^2}\right)\sqrt{\frac{W/S_W}{\rho}\left(1+\frac{1}{k_g^2}\right)C_{V,ave}}. \tag{10}$$

From Eqs. (9) and (10),

$$V_d=\sqrt{\frac{W/S_W}{\rho}}\left\{\sqrt{\frac{2}{DF}}+\frac{1}{2}\left(\frac{S_W}{\pi R^2}\right)\sqrt{\frac{DF}{2}}\right\}. \tag{11}$$